If the functions p and q are of constant sign and condition (3) (condition (4)) is violated, then there exist constants c_{i1} $(i=1,2)$ such that problem (1), (2_1) (problem (1), (2_2)) has no solution whatever c_{ik} $(i=1,2;\ k=2,\dots,m)$ are. Therefore conditions (3) and (4) in Theorems 1 and 2 are unimprovable.

As for condition (5), it is necessary for the unique solvability of problem (1), (2_2) since if $p(t)\equiv(-1)^m\left(\frac{\pi}{b-a}\right)^{2m}$, then the homogeneous problem

$$u^{(2m)}=p(t)u;\quad u^{(2k-2)}(a)=u^{(2k-2)}(b)=0\quad(k=1,\dots,m)$$

has a nontrivial solution $u(t)=\sin\left(\frac{\pi}{b-a}t\right)$.

THE TWO-POINT BOUNDARY VALUE PROBLEM FOR THE GENERALIZED MATRIX RICCATI EQUATION

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Consider a Riccati equation of the following form:

$$\frac{dX}{dt}=A_1(t)XA_2(t)+B_1(t)XB_2(t)+XQ(t)X+F(t)\equiv G(t,X),\tag{1}$$

where $X\in\mathbb{R}^{n\times n}$, $A_i,B_i(i=1,2),Q,F\in\mathbb{C}(I,\mathbb{R}^{n\times n})$, $I=[0,\omega]$, $\omega>0$.

We will study a two-point boundary-value problem for (1) in the finite-dimensional Banach algebra $\mathcal{B}(n)$ of continuous matrix functions with the norm $\|X\|_{\mathbb{C}}=\max\limits_t\|X(t)\|$ in case of

$$MX(0)+NX(\omega)=0,\tag{2}$$

where M and N are real $n\times n$ matrices.

The present work is a continuation of [1–4] and deals with a constructive analysis of problem (1), (2) on the basis of the method presented in [5, chapter 1; 6].

We introduce the following notations:

$$D_\rho=\{(t,X):t\in I,\|X\|\leqslant\rho\},\quad \alpha_i=\max_t\|A_i(t)\|,\quad \beta_i=\max_t\|B_i(t)\|(i=1,2),$$

$$h=\max_t\|F(t)\|,\quad \delta=\max_t\|Q(t)\|,\quad \mu=\max(\|M\|,\|N\|),\quad \gamma=\|(M+N)^{-1}\|,$$

$$\varphi(\rho)=a_0\rho^2+a_1\rho+a_2,\quad \rho_1=\frac{1-a_1-\sqrt{(1-a_1)^2-4a_0a_2}}{2a_0},\quad \rho^*=\frac{1-a_1}{2a_0},$$

where $\rho>0$, $t\in I$, $a_0=\gamma\mu\delta\omega$, $a_1=\gamma\mu(\alpha_1\alpha_2+\beta_1\beta_2)\omega$, $a_2=\gamma\mu\omega h$.

Lemma. *Let the following conditions be fulfilled:* $\det(M+N)\neq0$, $\varphi(\rho)\leqslant\rho$, $\varphi'(\rho)<$ <1. *Then the solution of problem* (1), (2) *exists in the region* D_ρ, *it is unique and can be* *presented as a uniform limit of a sequence of matrix functions determined by the recurrent integrated relationship*

$$X_{k+1}(t) = (M+N)^{-1}\Big\{M\int\limits_0^t G(\tau, X_k(\tau))\,d\tau - N\int\limits_t^\omega G(\tau, X_k(\tau))\,d\tau\Big\}, \quad k = 0,1,2,\dots \quad (3)$$

The matrix in (3) is arbitrary $\mathbb{C}(I, \mathbb{R}^{n\times n})$ – class matrix, which belongs to the sphere $\|X_0\|_{\mathbb{C}} \leqslant \rho$. Then the matrix function $X_m(t)(m = 1,2,\dots)$, it also satisfy to condition (2). Using condition $\varphi(\rho) \leqslant \rho$ and induction by k, one can show readily that members of sequence $X_k(t)_0^\infty$, belong to the sphere $\|X\|_{\mathbb{C}} \leqslant \rho$.

Theorem. *Let the Lemma conditions be fulfilled. Then the solution of problem* (1), (2) *exists in the region* D_ρ $(\rho_1 \leqslant \rho < \rho^*)$, *it is unique and can be presented as a uniform limit of a sequence of matrix functions determined by the recurrent integrated relationship* (3). *This solution satisfies the estimation*

$$\|X\|_{\mathbb{C}} \leqslant \rho_1. \quad (4)$$

Specified sequence converges uniformly in $t \in I$ to the solution of integral equation

$$X(t) = (M+N)^{-1}\Big\{M\int\limits_0^t G(\tau, X(\tau))\,d\tau - N\int\limits_t^\omega G(\tau, X(\tau))\,d\tau\Big\}. \quad (5)$$

Estimation (4) can be obtained a priori by (5).

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ON ONE GENERALIZATION OF THE LYAPUNOV–HARTMAN–WINTNER THEOREM

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A.M. Lyapunov has proved (see [1]) that if $p : [a,b] \to \mathbb{R}$ is a continuous function, satisfying the condition

$$\int\limits_a^b [p(t)]_-\,dt \leqslant \frac{4}{b-a}, \quad (1)$$