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Analysis of contact strength of spherical roller transmission with double-row pinion

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Abstract. This paper presents a detailed description of the design of the spherical roller transmission. The pinion has two rows of rollers; the axes of the rollers are located on conical surfaces. When the transmission is operating, the pinion performs a regular precession and the rollers are in contact with the teeth of the stationary and driven central gears. The transmission provides high reduction ratios in a lightweight, compact package. The center of mass of the pinion is located on the axis of rotation of the drive shaft, which reduces the axial moment of inertia and improves the balance of the system compared to double-pinion planetary gears mounted on the drive shaft with eccentricity. Rolling bodies are used in place of teeth, which leads to improved efficiency. The main performance criterion is the contact strength of the teeth of the driven central gear. An equation to calculate maximum contact stresses has been derived by using Hertz theory. An algorithm for calculating the number of the rollers simultaneously engaged and transferring the load is proposed.

1. Introduction

Power drives with high-speed motors and slowly rotating drive shafts find application in various fields: positioning and servo systems, kinematic chains of robots, the aerospace industry. Besides, they are widely used in household appliances. To achieve high reduction ratios in drives with a compact design, mechanical transmissions of various types are used. Simple planetary gears can produce reduction ratios up to 50 ... 60. However, their reliability and efficiency significantly depend on the way in which rotation is transmitted from the pinion to the driven shaft [1]. Due to the use of double-pinion planetary gears, the range of reduction ratios can be increased significantly; the disadvantages are increased slip and low efficiency [2]. Power losses can be reduced by replacing teeth with rolling bodies [3]. Transmissions with the pinion that does not move in a plane but performs a regular precession are referred to as nutation or precessional transmissions [4–7]. They have a lower total axial moment of inertia and smaller radial dimensions. Since the pinion performs a spherical motion, the transmissions fall within the category of spherical mechanisms, which can be used in various fields [8, 9]. To increase the number of rollers simultaneously transferring the load, a number of designs of spherical roller transmissions (SRT) with rows of rollers arranged coaxially have been developed [10]. To reduce radial dimensions of the transmission, the rows of rollers in most precessional transmissions are arranged axially relative to each other [11]. This paper presents a description of a new SRT design developed to increase the transmitted unit load. The study is aimed to develop a new transmission design and estimate the strength of the roller engagement.



2. SRT model and principle of operation

Figure 1 shows the main elements of the SRT 3D model created in NX and intended for subsequent kinematic and strength calculations. The rollers are combined with a pinion and are made as cylindrical teeth.

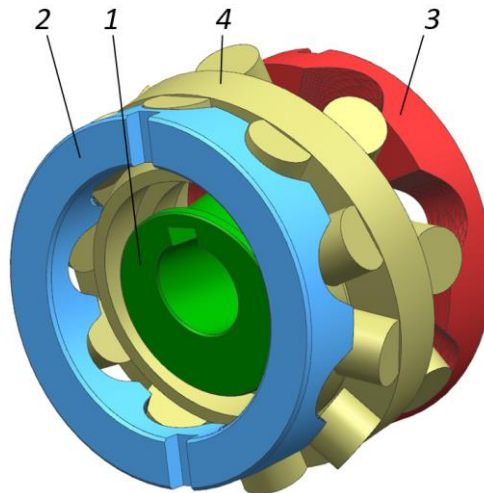


Figure 1. SRT model (main elements):
1 – eccentric; 2 – driven central gear;
3 – stationary central gear; 4 – pinion.

The drive shaft of SRT has a section inclined to its axis at a nutation angle Θ . The inclination is provided by means of eccentric 1 installed on the drive shaft. Pinion 4 is mounted on the eccentric by means of bearings. The rollers are arranged in two rows with uniform angular spacing; their axes in each row are located on the conical surface and are symmetrical with respect to the axis of rotation of the pinion. One row of the rollers is in contact with the teeth of stationary central gear 3, which is fixed in the housing, and the second row of the rollers engages with the teeth of central gear 2, which is connected to the driven shaft. The teeth of the central gears have a constant height. The rollers, alternately getting into the extreme left and right positions where they extend into tooth spaces of the gears to the maximum depth, occupy positions where their axes are perpendicular to the axes of the transmission. In this way, the maximum length of the contact lines and the transmitted unit load are reached. Due to the fact that the rollers can rotate about their axes, sliding in contact is partially replaced by rolling, which leads to increased efficiency of the transmission.

Equations for determining the reduction ratio for SRT with a double-row pinion are similar to those for double-pinion planetary gears. The number of rollers in rows performs the function of the number of gear teeth. The range of produced reduction ratios can be estimated as 16 ... 200 for transmissions with diametrical dimensions not exceeding 250 mm.

The main reliability criterion for SRT is the contact strength of the transmission parts [12]. To perform the check calculation, the following data must be taken into account: the nominal torque on the driven shaft T_2 , N·m, materials used to make parts and heat treatment techniques, the reduction ratio, the number of teeth on the driven gear Z_2 and the stationary gear Z_3 , respectively, the number of rollers in the pinion rows n_{s2} and n_{s3} , respectively, as well as geometric parameters of the transmission.

3. SRT contact strength analysis

Let us consider the interaction of one row of rollers with the driven central gear, as it is subjected to higher loads. The middle section is taken as the section for strength calculations, the parameters are assigned the subscript m (figure 2). Space curve 4 (center curve) is the trajectory of point C lying on

the axis of roller 1 when it moves relative to the driven central gear.

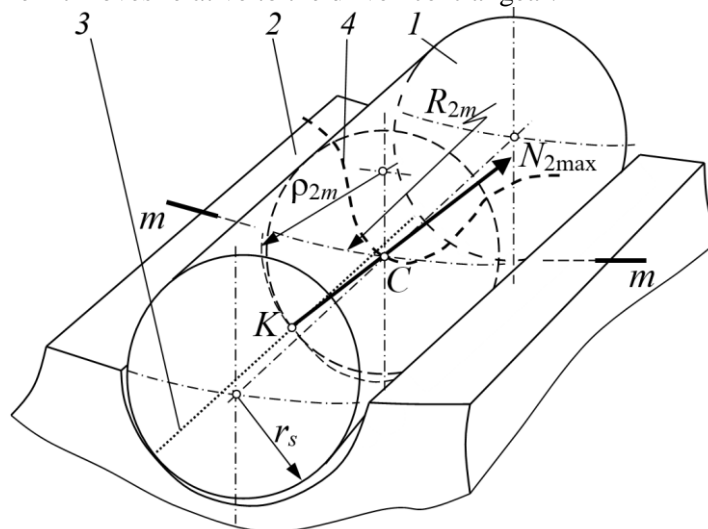


Figure 2. Schematic representation of roller contact with central gear teeth: 1 – roller; 2 – toothed profile; 3 – contact line; 4 – center curve in the middle section.

Maximum contact stresses $\sigma_{H_{\max}}$ are calculated using the formula modified for interaction of a cylinder and a concave surface [13] according to Hertz theory

$$\sigma_{H_{\max}} = K_{Gs} \cdot \sqrt{N_{2\max} \cdot \frac{\rho_{2m} - r_s}{\rho_{2m} \cdot r_s}}, \quad (1)$$

where K_{Gs} is the coefficient depending on the properties of the materials of contacting parts, $\text{MPa}^{1/2}$; $N_{2\max}$ is the maximum value of the normal force in the contact between the roller and the teeth, N; ρ_{2m} and r_s are the radii of curvature of the contacting surfaces (the toothed profile in the middle section and the roller, respectively), mm.

The coefficient K_{Gs} is calculated using the formula

$$K_{Gs} = 0.564 \cdot (\theta_1 + \theta_2)^{-0.5}, \quad (2)$$

where θ_1 and θ_2 are the coefficients depending on the properties of materials of contacting bodies.

For steel parts $K_{Gs} = 473.6 \text{ MPa}^{1/2}$. Then

$$\theta_{1(2)} = (1 - \mu_{1(2)}^2) \cdot E_{1(2)}^{-1}, \quad (3)$$

where $\mu_{1(2)}$ is the Poisson's ratio of the material of roller (1) and the teeth of driven central gear (2), respectively; $E_{1(2)}$ is the elastic modulus. For steel parts $E_1 = E_2 = 2.1 \cdot 10^5 \text{ MPa}$, $\mu_1 = \mu_2 = 0.3$.

To calculate the radius of curvature, let us consider the scheme of the middle section of the toothed profile in figure 3. We approximate the complex profile of surface 1 replacing it with circular arc 2. From geometric constructions, using the properties of the chords of the circle, we obtain

$$\rho_{2m} = 2 \cdot R_m^2 \cdot \sin^2(\psi_{z0}) \cdot r_s^{-1} + 0.5 \cdot r_s, \quad (4)$$

where R_m is the radius of the basic sphere with the center curve in the middle section, mm; ψ_{z0} is the central angle, rad, calculated using the formula

$$\psi_{z0} = \frac{\pi \cdot r_s}{R_m \cdot Z_2 \cdot \sin(\alpha_{2m})}, \quad (5)$$

where α_{2m} is the mean value of the slope of the center curve determined in the middle section of the driven central gear.

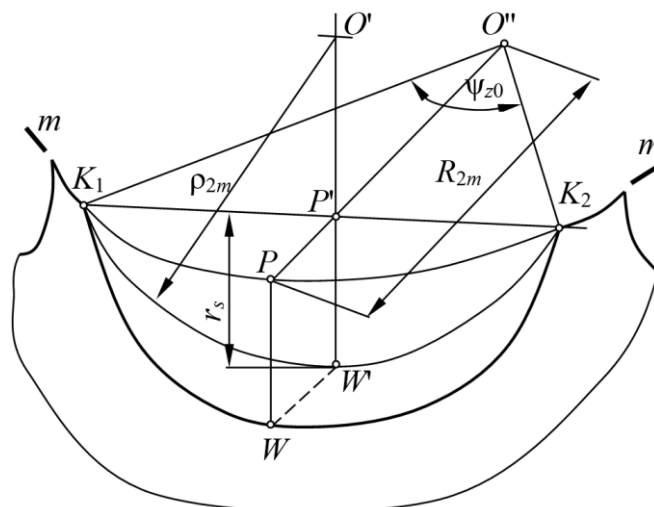


Figure 3. Scheme for determining radius of toothed profile curvature in middle section.

To calculate the slope of the center curve, we simplify the problem by replacing it with a piecewise helix and studying its unfolded view. The resulting curve with constant values of slopes is a set of straight line segments; the slopes are calculated according to the formula

$$\alpha_{2m} = \arctg(2 \cdot Z_2 \cdot \Theta \cdot \pi^{-1}), \quad (6)$$

where R_{2m} is the radius of the circle with the point K of application of the normal force, mm.

The maximum normal load on one roller, N , is calculated as follows

$$N_{2\max} = \frac{T_2 \cdot 10^3}{R_{2m} \cdot n_{s2} \cdot K_{p2} \cdot K_n \cdot \sin(\alpha_{2m})}, \quad (7)$$

where n_{s2} is the number of the rollers in the pinion row interacting with the driven central gear; K_{p2} is the coefficient that takes into account the number of the rollers simultaneously transferring the load; K_n is the coefficient that takes into account uneven load distribution through the flows.

The scheme for determining the coefficient K_{p2} (figure 4, a) shows the roller in the leftmost position with the axis in a vertical position. As the transmission continues to run, the roller starts receding from the contact with the toothed profile of the flat gear, and the lowest leftmost point B of the roller is the last to move out of engagement. By connecting these points for all the rollers in the row, we obtain circle 2 (figure 4, b). Central gear 4 is bounded on the right by plane 1. Let us suppose that the rollers with the points which lie furthest to the left of plane 1, i.e., “penetrating” into the toothed profile, engage with the gear, and the part of the circle to the right of this plane unites the leftmost points of the rollers that are not in contact with the gear teeth.

Then the coefficient K_{p2} , which is determined as the number of the rollers simultaneously transferring the load relative to the total number of the rollers, can be calculated as the ratio of the angle φ_r to the angle $2 \cdot \pi$. The angle φ_r is the operating angle and the angle equal to $2 \cdot \pi - \varphi_r$ is the idling angle.

To find the operating angle, let us first calculate the angle φ_s using the parametric equation of circle 1 derived earlier, which describes the change in the z -coordinate depending on the parameter φ , assuming that in the equation $z(\varphi) = L_s$ (figure 4, b). After transformations we obtain

$$\sin(\varphi_s) = \frac{L_s - (L_{k\max} + r_s \cdot \cos(\Theta)) \cdot \cos(\Theta)}{\sin(\Theta) \cdot \sqrt{R_{2\min}^2 + (L_s + r_s)^2 - (L_{k\max} + r_s \cdot \cos(\Theta))^2 \cdot \cos^2(\Theta)}}, \quad (8)$$

where $L_{k\max}$ and $R_{2\min}$ are the geometric parameters shown in figure 4, a.

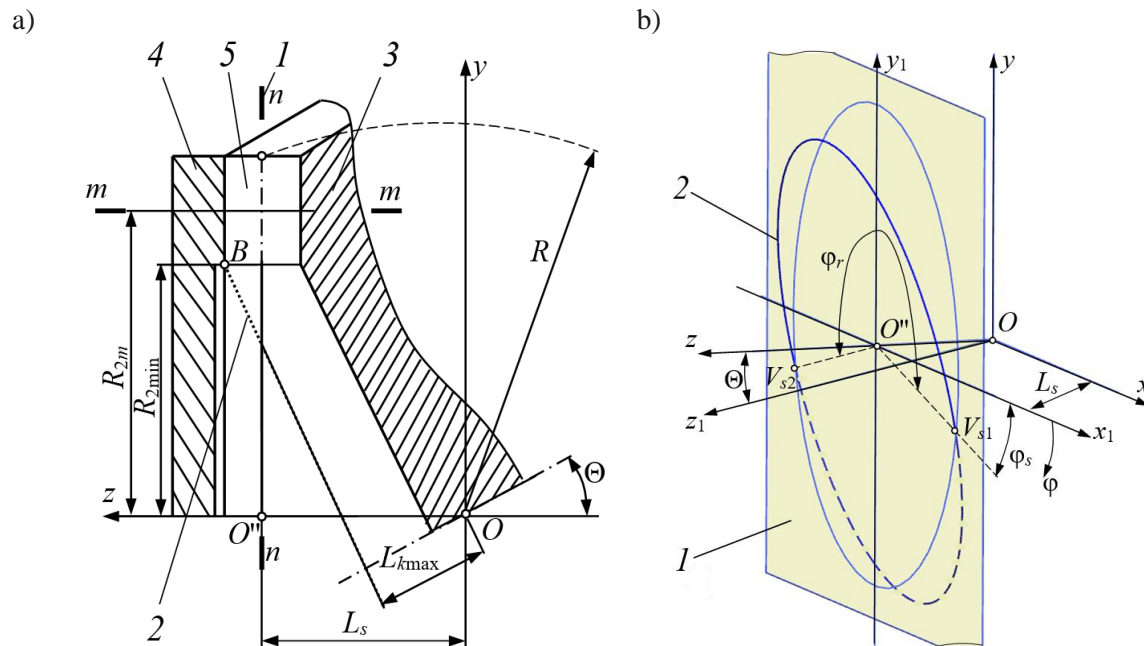


Figure 4. Contact ratio calculation: a – schematic representation of the contact; b – computational scheme; 1 – plane bounding the toothed profile; 2 – circle with the lowest leftmost points of rollers; 3 – pinion; 4 – driven central gear; 5 – roller.

Finally, the contact strength condition is written as follows:

$$\sigma_{H \max} = K_{Gs} \cdot \sqrt{\frac{T_2 \cdot 10^3}{R_{2m} \cdot n_{s2} \cdot K_{p2} \cdot K_n \cdot \sin(\alpha_{2m})} \cdot \frac{\rho_{2m} - r_s}{\rho_{2m} \cdot r_s}} \leq [\sigma_H], \quad (9)$$

where $[\sigma_H]$ are the allowable contact stresses, MPa.

Using the above formulas, a check calculation for the transmission with a reduction ratio of 36 and the number of teeth and rollers of $Z_2 = 9$, $Z_3 = 7$, $n_{s2} = 10$, $n_{s3} = 8$ was performed. The following parameters were used in the calculation: the torque on the driven shaft $T_2 = 50 \text{ N}\cdot\text{m}$, the radius of the basic sphere determining the dimensions of the transmission $R = 30 \text{ mm}$ (figure 4, a), $r_s = 5 \text{ mm}$ and $\Theta = 0.2 \text{ rad}$. The transmission parts were made of steel with accuracy grade 7; the experimental coefficient was taken to be $K_n = 0.9$. According to the results of the calculations performed using Mathcad, the maximum values of contact stress were determined to be $\sigma_{H \max} = 605 \text{ MPa}$. The coefficient taking into account the proportion of the rollers engaged simultaneously was found to be $K_{p2} = 0.538$.

4. Conclusions

On the basis of Hertz theory, the equation which can be used to calculate maximum contact stresses in the teeth of the SRT central driven gear has been derived. Allowable stresses, equal to or greater than the calculated values, can be obtained by using the most common and relatively low-cost types of alloy steels and appropriate heat treatment techniques. The algorithm for calculating the number of rollers simultaneously transferring the load to SRT has been developed. The calculations performed using this algorithm have shown that more than half of the rollers of the transmission with the above parameters are simultaneously engaged and their ratio to the total number of the rollers does not depend on the reduction ratio. The analysis of the developed model shows that with the same geometric parameters of the transmission specified above, the maximum contact stresses decrease as the reduction ratio increases, which is due to the greater number of teeth on the central gears and rollers. However, the number of teeth does not exceed some limiting value subject to the condition $\rho_{2m} \geq r_s$.

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