задача (1), (2) однозначно разрешима в соболевском пространстве функций $W^{2,4}_{2,\gamma}(R^{n+1}_{++})$, $\gamma > \gamma_0$, *таких, что* $D^2_t D^2_{x_i} u(t,x) \in L_{2,\gamma}(R^{n+1}_{++})$, *и для решения* $u(t,x)$ *выполняется оценка:*

$$\|u(t,x), W^{2,4}_{2,\gamma}(R^{n+1}_{++})\| \leq c(\gamma_0)\|f(t,x), W^{1,0}_{2,\gamma}(R^{n+1}_{++})\|,$$

где константа $c(\gamma_0)$ *не зависит от* $f(t,x)$.

Теорема 2. *Существует* $\gamma_0 > 0$ *такое, что для любой*

$$f(t,x) \in W^{0,1}_{2,\gamma}(R^{n+1}_{++}), \quad \gamma > \gamma_0,$$

задача (1), (2) однозначно разрешима в соболевском пространстве функций $W^{2,4}_{2,\gamma}(R^{n+1}_{++})$, $\gamma > \gamma_0$, *таких, что* $D^2_t D^2_{x_i} u(t,x) \in L_{2,\gamma}(R^{n+1}_{++})$, *и для решения* $u(t,x)$ *выполняется оценка:*

$$\|u(t,x), W^{2,4}_{2,\gamma}(R^{n+1}_{++})\| \leq c(\gamma_0)\|f(t,x), W^{0,1}_{2,\gamma}(R^{n+1}_{++})\|,$$

где константа $c(\gamma_0)$ *не зависит от* $f(t,x)$.

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COMPARISON PRINCIPLE FOR INITIAL BOUNDARY VALUE PROBLEM FOR NONLINEAR NONLOCAL PARABOLIC EQUATION

A.L. Gladkov

Let $Q_T = \Omega \times (0,T)$, $S_T = \partial\Omega \times (0,T)$, $\Gamma_T = S_T \cup \overline{\Omega} \times \{0\}$, $T > 0$.

We consider the initial boundary value problem for nonlinear nonlocal parabolic equation

$$u_t = \Delta u + a u^p \int_\Omega u^q(y,t)\,dy - b u^m, \ (x,t) \in Q_T, \tag{1}$$

with nonlinear nonlocal boundary condition

$$\frac{\partial u(x,t)}{\partial \nu} = \int_\Omega k(x,y,t) u^l(y,t)\,dy, \ (x,t) \in S_T, \tag{2}$$

and initial datum

$$u(x,0) = u_0(x), \ x \in \Omega, \tag{3}$$

where a, b, p, q, m, l are positive numbers, Ω is a bounded domain in $\mathbb{R}^N$ for $N \geq 1$ with smooth boundary $\partial\Omega$, ν is unit outward normal on $\partial\Omega$.

We suppose that the functions $k(x,y,t)$ and $u_0(x)$ satisfy the following conditions:

$$k(x,y,t) \in C(\partial\Omega \times \overline{\Omega} \times [0,+\infty)),\ k(x,y,t) \geq 0;$$

$$u_0(x) \in C^1(\overline{\Omega}),\ u_0(x) \geq 0 \text{ in } \Omega,\ \frac{\partial u_0(x)}{\partial \nu} = \int\limits_{\Omega} k(x,y,0) u_0^l(y)\, dy \text{ on } \partial\Omega.$$

Initial boundary value problem for parabolic equation (1) with nonlocal boundary condition

$$u(x,t) = \int\limits_{\Omega} k(x,y,t) u^l(y,t)\, dy,\ (x,t) \in S_T$$

was considered in [1, 2].

Definition. We say that a nonnegative function $u(x,t) \in C^{2,1}(Q_T) \cap C^{1,0}(Q_T \cup \Gamma_T)$ is a supersolution of (1)–(3) in Q_T if

$$u_t \geq \Delta u + a u^p \int\limits_{\Omega} u^q(y,t)\, dy - b u^m,\ (x,t) \in Q_T, \tag{4}$$

$$\frac{\partial u(x,t)}{\partial \nu} \geq \int\limits_{\Omega} k(x,y,t) u^l(y,t)\, dy,\ x \in \partial\Omega,\ 0 < t < T, \tag{5}$$

$$u(x,0) \geq u_0(x),\ x \in \Omega, \tag{6}$$

and $u(x,t) \in C^{2,1}(Q_T) \cap C^{1,0}(Q_T \cup \Gamma_T)$ is a subsolution of (1)–(3) in Q_T if $u \geq 0$ and it satisfies (4)–(6) in the reverse order. We say that $u(x,t)$ is a solution of problem (1)–(3) in Q_T if $u(x,t)$ is both a subsolution and a supersolution of (1)–(3) in Q_T.

Theorem 1. *Let $\overline{u}$ and $\underline{u}$ be a supersolution and a subsolution of problem (1)–(3) in Q_T, respectively. Suppose that $\underline{u}(x,t) > 0$ or $\overline{u}(x,t) > 0$ in $Q_T \cup \Gamma_T$ if $\min(p,q,l) < 1$. Then $\overline{u}(x,t) \geq \underline{u}(x,t)$ in $Q_T \cup \Gamma_T$.*

Remark. We improve a comparison principle in [3]. The authors of [3] supposed that $\underline{u}(x,t) > 0$ or $\overline{u}(x,t) > 0$ in $Q_T \cup \Gamma_T$ if $\min(m,p,q,l) < 1$.

Theorem 2. *Let $u_0 \not\equiv 0$ in Ω, $m \geq 1$. Suppose u is a solution of (1)–(3) in Q_T. Then $u > 0$ in $Q_T \cup S_T$.*

The results of the talk have been published in [4].

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