8. Polovinkin I. P. *Mean Value Theorems for Linear Partial Differential Equations* // J. Math. Sci. 2014. Vol. 197. № 3. P. 399–403.

9. Kuznetsov N. *Mean Value Properties of Solutions to the Helmholtz and Modified Helmholtz Equations* // J. Math. Sci. 2021. Vol. 257. № 5. P. 673–683.

10. Kuznetsov N. *Asymptotic Mean Value Properties of Meta- and Panharmonic Functions* // J. Math. Sci. 2021. Vol. 259. № 2. P. 205–209.

11. Polovinkin I. P., Polovinkina M. V. *Mean Value Theorems and Properties of Solutions of Linear Differential Equations.* In: Transmutation Operators and Applications. Trends in Mathematics. (Eds. V. Kravchenko, S. Sitnik) Cham, Birkhäuser, 2020. P. 587–602.

12. Pokrovskii A. V. *Mean Value Theorems for Solutions of Linear Partial Differential Equations* // Mathematical Notes. 1998. Vol. 64. № 2. P. 220–229.

13. Hörmander L. *Asgeirsson's Mean Value Theorem and Related Identities* // Journal of Functional Analysis. 2001. Vol. 184. № 2. P. 377–401.

REDUCTION OF SOME EVOLUTIONARY EQUATIONS BY MEANS OF SYMMETRIES

D. S. Zhalukevich

Reversible transformations for partial differential equations in the case of two independent variables t, x and one dependent variable $u = u(t,x)$, have the form [1-4]

$$\widetilde{t} = \varphi(t,x,u,\varepsilon), \quad \widetilde{x} = \psi(t,x,u,\varepsilon), \quad \widetilde{u} = \chi(t,x,u,\varepsilon), \tag{1}$$

where

$$\varepsilon \in \mathbb{R},$$

with

$$\varphi(t,x,u,0) = t, \quad \psi(t,x,u,0) = x, \quad \chi(t,x,u,0) = u. \tag{2}$$

The construction of the symmetry group is equivalent to the definition of its infinitesimal transformations

$$\widetilde{t} = t + \varepsilon\tau(t,x,u) + ..., \quad \widetilde{x} = x + \varepsilon\xi(t,x,u) + ..., \quad \widetilde{u} = u + \varepsilon\eta(t,x,u) + ..., \tag{3}$$

where

$$\tau(t,x,u) = \frac{\partial\varphi(t,x,u,0)}{\partial\varepsilon}, \quad \xi(t,x,u) = \frac{\partial\psi(t,x,u,0)}{\partial\varepsilon}, \quad \eta(t,x,u) = \frac{\partial\chi(t,x,u,0)}{\partial\varepsilon}. \tag{4}$$

Consider the k-th order evolutionary equations:

$$u_t + A_0(t,x,u) + \sum_{i=1}^{k} A_i(t,x,u)u_{ix} = 0, \tag{5}$$

where

$$u_{ix} = \frac{\partial^i u}{\partial x^i}, \quad k = 2,3,4.$$

Theorem. *For equation (5), the symmetry transformations (1) have the form*

$$\widetilde{t} = \varphi(t,\varepsilon), \quad \widetilde{x} = \psi(t,x,\varepsilon), \quad \widetilde{u} = \chi(t,x,u,\varepsilon), \tag{6}$$

this means that one can search for infinitesimal symmetries in the form

$$X = \tau(t)\frac{\partial}{\partial t} + \xi(t,x)\frac{\partial}{\partial x} + \eta(t,x,u)\frac{\partial}{\partial u}, \tag{7}$$

where

$$\eta = b(t,x)u + c(t,x). \tag{8}$$

Among the equations (5), the following equations are found that can be reduced [5-23]:

$$u_t + u^n u_x + f(t,x)u_{kx} = 0, \quad n \in \mathbb{N}, \tag{9}$$

$$u_t + u^n u_x + (f(t,x)u)_{kx} = 0, \quad n \in \mathbb{N}, \tag{10}$$

$$u_t + u^n u_x + f(t,x)u_{xx} + g(t,x)u_{xxx} + h(t,x)u_{xxxx} = 0, \tag{11}$$

$$u_t + u^n u_x + (f(t,x)u)_{xx} + (g(t,x)u)_{xxx} + (h(t,x)u)_{xxxx} = 0, \tag{12}$$

$$u_t + q(t,x)u^n u_x + u_{kx} = 0, \quad n \in \mathbb{N}, \tag{13}$$

$$u_t + (q(t,x)u^{n+1})_x + u_{kx} = 0, \quad n \in \mathbb{N}. \tag{14}$$

The equation (9) for $n = 1$ and $k = 2$ is called the homogeneous Burgers equation; for $n > 1$ and $k = 2$, the homogeneous convection-diffusion equation; for $n = 1$ and $k = 3$, the homogeneous Korteweg-de equation For $n > 1$ and $k = 3$ homogeneous modified Korteweg-De Vries equation; for $n = 1$ and $k = 4$ homogeneous generalized Burgers-Korteweg-de Vries equation; for $n > 1$ and $k = 4$ homogeneous generalized modified the Burgers-Korteweg-de Vries equation.

The equation (11) for $n = 1$ and $h = 0$ is called the homogeneous Burgers-Korteweg-de Vries equation; for $n > 1$ and $h = 0$, the homogeneous modified Burgers-Korteweg-De Vries equation; for $n = 1$, the homogeneous equation Kuramoto-Sivashinsky; for $n > 1$ by a homogeneous modified Kuramoto-Sivashinsky equation.

Refrences

1. Hydon P.E. *Symmetry Methods for Differential Equations: A Beginner's Guide* Cambridge University Press, 2000.

2. Ibragimov N.H. *Transformation Groups Applied to Mathematical Physics*. Reidel, Dordrecht, 1985.

3. Ovsiannikov L.V. *Group Analysis of Differential Equations* Academic, New York, 1982.

4. Olver P.J. *Applications of Lie Groups to Differential Equations* Springer-Verlag, New York, 1993, 2nd edition, Graduate Texts Math., vol. 107.

5. Clarkson P.A., Winternitz, P. *Symmetry Reduction and Exact Solutions of Nonlinear Partial Differential Equations*. In: Conte, R. (eds) The Painleve Property. CRM Series in Mathematical Physics. Springer, New York, NY, 2002.

6. Opanasenko S., Bihlo A., Popovych R. *Group analysis of general Burgers–Korteweg–de Vries equations* // Journal of Mathematical Physics, 2017.

7. Clarkson P.A. *Dimensional reductions and exact solutions of a generalized nonlinear Schrodinger equation* // Nonlinearity. 1992. 5. 453-472.

8. Clarkson P.A. *Nonclassical symmetry reductions of nonlinear partial differential equations* // Math. Comp. Model. 1993. 18. 45-68.

9. Clarkson P.A. *Nonclassical symmetry reductions of the Boussinesq equation* // Chaos, Solitons and Fractals. 1995. 5. 2261-230l.

10. Clarkson P.A., Hood S. |it Nonclassical symmetry reductions and exact solutions of the Zabalotskaya-Khokhlov equation. // Europ. J. Appl. Math. 1992. 3. 381-414.

11. Clarkson P.A., Ludlow D.K, Priestley T.J. *The classical, direct and nonclassical methods for symmetry reductions of nonlinear partial differential equations* // Meth. Appl. Anal. 1997. 4. 173-195.

12. Clarkson P.A., Mansfield E.L. *Symmetry Reductions and Exact Solutions of a class of Nonlinear Heat Equations* // Physica. 1994. D 70. 250-288.

13. Clarkson P.A., Mansfield E.L. *Algorithms for the nonclassical method of symmetry reductions* // SIAM J. Appl. Math. 1994. 54. 1693-1719.

14. Clarkson P.A. and Mansfield E.L. *Symmetry reductions and exact solutions of shallow water wave equations* // Acta Appl. Math. 1995. 39. 245-276.

15. Clarkson P.A., Winternitz P. *Nonclassical symmetry reductions for the Kadomtsev-Petviashvili equation* // Physica. 1991. D 49. 257-272.

16. Levi D., Winternitz P. *Nonclassical symmetry reduction: example of the Boussinesq equation* // J. Phys. 1989. A 22. 2915-2924.

17. Lou S.-Y. *A note on the new similarity reductions of the Boussinesq equation* // Phys. Lett. 1990. l5lA. 133-135.

18. Ludlow D.K. *Nonclassical similarity reductions of the Navier-Stokes equations* Ph.D. thesis (Department of Mathematics, University of Exeter, UK), 1994.

19. Mansfield E.L. , Reid G.J., ClarksonP.A. *Nonclassical reductions of a coupled nonlinear Schrodinger system // Compo Phys. Comm. 1999. 115. 460488.*

20. Nucci M.C., Clarkson P.A. The nonclassical method is more general than the direct method for symmetry reductions: an example of the Fitzhugh-Nagumo equation // Phys. Lett. 1992. 164A. 49-56.

21. Olver P.J. Direct reduction and differential constraints // Proc. R. Soc. Lond. 1994. A 444. 509-523.

22. Pucci E. Similarity reductions of partial differential equations // J. Phys. 1992. A 25. 2631-2640.

23. Zidowitz S., Conditional symmetries and the direct reduction of partial differential equations, Modern Group Analysis: Advanced Analytical and Computational Methods in Mathematical Physics, eds. N.H. Ibragimov, M. Torrisi, and A. Valenti. Kluwer, Dordrecht. 1993. P. 387-393.