Refrences

1. Kisil A. V., Abrachams I. D., Mishuris G., Rogosin S. V. *The Wiener-Hopf technique, its generalizations and applications: constructive and approximate methods* // Proc. R. Soc. A. 2021. Vol. 477. № 2254. Paper No. 20210533. 32 pp.

2. Rogosin S., Mishuris G. *Constructive methods for factorization of matrix-functions* // IMA Journal of Applied Mathematics. 2016. Vol. 81. №2. P. 365–391.

3. Chebotarev, G. N. *Partial indices of the Riemann boundary value problem with triangular matrix of the second order*// Uspekhi mat. nauk. 1956. Vol. XI. № 3. 192–202 (in Russian)

4. Primachuk L., Rogosin S. *Factorization of triangular matrix-functions of an arbitrary order* // Lobachevskii Journal of Mathematics. 2018. Vol. 39. № 1. P. 129–137.

5. Primachuk L. P., Rogosin S. V. and Dubatovskaya M. V. *On factorization of partly non-rational* 2×2 *matrix-functions* // Transactions of A. Razmadze Mathematical Institute. 2022. Vol. 176. № 3. P. 403–410.

MULTI-DIMENSIONAL INTEGRAL TRANSFORMATION WITH FOX H-FUNCTION IN THE KERNEL IN THE WEIGHTED SPACES OF SUMMABLE FUNCTIONS

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Multidimensional integral transform

$$\big(\mathrm{H}f\big)(\mathbf{x})=\int\limits_{\mathbf{0}}^{\infty}\mathrm{H}^{\mathbf{m},\mathbf{n}}_{\mathbf{p},\mathbf{q}}\left[\mathbf{xt}\,\middle|\,\begin{matrix}(\mathbf{a}_{\mathrm{i}},\overline{\alpha}_{\mathrm{i}})_{1,\mathrm{p}}\\(\mathbf{b}_{\mathrm{j}},\overline{\beta}_{\mathrm{j}})_{1,\mathrm{q}}\end{matrix}\right]\mathrm{f}(\mathbf{t})\mathbf{dt}\ \ (\mathbf{x}>0) \tag{1}$$

is studied. Here (see [1]; [2]; [3], Section 28.4; [4], ch. 1; [5]; [6]) $\mathbf{x}=(x_1,x_2,...,x_n)\in\mathrm{R}^{\mathrm{n}}$; $\mathbf{t}=(t_1,t_2,...,t_n)\in\mathrm{R}^{\mathrm{n}}$, R^{n} be the n-dimensional Euclidean space; $\mathbf{x}\cdot\mathbf{t}=\sum\limits_{n=1}^{n}x_nt_n$ denotes their scalar product; in particular, $\mathbf{x}\cdot\mathbf{1}=\sum\limits_{n=1}^{n}x_n$ for $\mathbf{1}=$ (1,1,...,1). The expression $\mathbf{x}>\mathbf{t}$ means that $x_1>t_1,x_2>t_2,...,x_n>t_n$, the nonstrict inequality $\geq$ has similar meaning ; $\int\limits_{\mathbf{0}}^{\infty}=\int\limits_{\mathbf{0}}^{\infty}\int\limits_{\mathbf{0}}^{\infty}\cdots\int\limits_{\mathbf{0}}^{\infty}$; by $\mathrm{N}=\{1,2,...\}$ we denote the set of positive integers, $\mathrm{N}_0=\mathrm{N}\bigcup\{0\}$, $\mathrm{N}_0^{\mathrm{n}}=\mathrm{N}_0\times\mathrm{N}_0\times...\times\mathrm{N}_0$; $\mathbf{k}=(k_1,k_2,...,k_n)\in\mathrm{N}_0^{\mathrm{n}}=\mathrm{N}_0\times...\times\mathrm{N}_0$ $(k_i\in\mathrm{N}_0,\ \mathrm{i}=1,2,...,\mathrm{n})$ is a multi-index with $\mathbf{k}!=k_1!\cdots k_n!$ and $|\mathbf{k}|=k_1+k_2+...+k_n$; $\mathrm{R}^{\mathrm{n}}_{+}=\{\mathbf{x}\in\mathrm{R}^{\mathrm{n}},\ \mathbf{x}>0\}$; for $l=(l_1,l_2,...,l_n)\in\mathrm{R}^{\mathrm{n}}_{+}$ $\mathbf{D}^l=\frac{\partial^{|l|}}{(\partial x_1)^{l_1}\cdots(\partial x_n)^{l_n}}$; $\mathbf{dt}=dt_1\cdot dt_2\cdots dt_n$; $\mathbf{t}^l=t^{l_1}t^{l_2}\cdots t^{l_n}$; $f(\mathbf{t})=f(t_1,t_2,...,t_n)$. Let C^{n} $(n\in\mathrm{N})$ be the n-dimensional space of n complex numbers $z=(z_1,z_2,\cdots,z_n)$ $(z_j\in\mathrm{C},\ j=1,2,\cdots,n)$; $\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}=\frac{d}{dx_1\cdot dx_2\cdots dx_n}$;

$\mathbf{m}=(m_1,m_2,...,m_n)\in\mathrm{N}_0^{\mathrm{n}}$ and $m_1=m_2=...=m_n$; $\mathbf{n}=(\overline{n}_1,\overline{n}_2,...,\overline{n}_n)\in\mathrm{N}_0^{\mathrm{n}}$ and $\overline{n}_1=\overline{n}_2=...=\overline{n}_n$; $\mathbf{p}=(p_1,p_2,...,p_n)\in\mathrm{N}_0$ and $p_1=p_2=...=p_n$; $\mathbf{q}=(q_1,q_2,...,q_n)\in\mathrm{N}_0$ and $q_1=q_2=...=q_n)$ $(0\leq\mathbf{m}\leq\mathbf{q},\ 0\leq\mathbf{n}\leq\mathbf{p})$;

$\mathbf{a}_i=(a_{i_1},a_{i_2},...,a_{i_n})$, $1\leq i\leq p$, $a_{i_1},a_{i_2},...,a_{i_n}\in\mathrm{C}$ $(1\leq i_1\leq p_1,...,1\leq i_n\leq p_n)$;

$\mathbf{b}_j=(b_{j_1},b_{j_2},...,b_{j_n})$, $1\leq j\leq q$, $b_{j_1},b_{j_2},...,b_{j_n}\in\mathrm{C}$ $(1\leq j_1\leq q_1,...,1\leq j_n\leq q_n)$;

$\overline{\alpha}_i=(\alpha_{i_1},\alpha_{i_2},...,\alpha_{i_n})$, $1\leq i\leq p$, $\alpha_{i_1},\alpha_{i_2},...,\alpha_{i_n}\in\mathrm{R}^{+}_{1}$ $(1\leq i_1\leq p_1,...,1\leq i_n\leq p_n)$;

$\overline{\beta}_j=(\beta_{j_1},\beta_{j_2},...,\beta_{j_n})$, $1\leq j\leq q$, $\beta_{j_1},\beta_{j_2},...,\beta_{j_n}\in\mathrm{R}^{+}_{1}$ $(1\leq j_1\leq q_1,...,1\leq j_n\leq q_n)$.

The function $\mathrm{H}^{\mathbf{m},\mathbf{n}}_{\mathbf{p},\mathbf{q}}\left[\mathbf{xt}\,\middle|\,\begin{matrix}(\mathbf{a}_{\mathrm{i}},\overline{\alpha}_{\mathrm{i}})_{1,\mathrm{p}}\\(\mathbf{b}_{\mathrm{j}},\overline{\beta}_{\mathrm{j}})_{1,\mathrm{q}}\end{matrix}\right]$ in the kernel of the (1) is the product of one type H-functions $H^{m,n}_{p,q}[z]$ [7, Chapters 1 and 2]:

$$\mathrm{H}^{\mathbf{m},\mathbf{n}}_{\mathbf{p},\mathbf{q}}\left[\mathbf{xt}\,\middle|\,\begin{matrix}(\mathbf{a}_{\mathrm{i}},\overline{\alpha}_{\mathrm{i}})_{1,\mathrm{p}}\\(\mathbf{b}_{\mathrm{j}},\overline{\beta}_{\mathrm{j}})_{1,\mathrm{q}}\end{matrix}\right]=\prod_{\mathrm{k}=1}^{\mathrm{n}}\mathrm{H}^{\mathrm{m_k},\,\overline{\mathrm{n}}_{\mathrm{k}}}_{\mathrm{p_k},\,\mathrm{q_k}}\left[\mathrm{x_k t_k}\,\middle|\,\begin{matrix}(\mathrm{a}_{\mathrm{i_k}},\alpha_{\mathrm{i_k}})_{1,\mathrm{p_k}}\\(\mathrm{b}_{\mathrm{j_k}},\beta_{\mathrm{j_k}})_{1,\mathrm{q_k}}\end{matrix}\right].$$

Our paper is devoted to the study of transform (1) in the weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{\mathfrak{r}}}$ - summable functions $f(\mathbf{x}) = f(x_1, x_2, ..., x_n)$ on $\mathrm{R}^{\mathbf{n}}_{+} = \{\mathbf{x} : \mathbf{x} \in \mathrm{R}^n | \;\; x_1 > 0, x_2 > 0, ..., x_n > 0\}$, such that

$$\|f\|_{\overline{\nu},\overline{r}} = \{\int\limits_{R^1_+} x_n^{\nu_n \cdot r_n - 1}\{\cdots\{\int\limits_{R^1_+} x_2^{\nu_2 \cdot r_2 - 1}$$

$$\times[\int\limits_{R^1_+} x_1^{\nu_1 \cdot r_1 - 1}|f(x_1, ..., x_n)|^{r_1} dx_1]^{r_2/r_1} dx_2\}^{r_3/r_2} \cdots\}^{r_n/r_{n-1}} dx_n\}^{1/r_n} < \infty$$

($\overline{r} = (r_1, r_2, ..., r_n) \in \mathrm{R}^n$, $1 \leq \overline{r} < \infty$, $\overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathrm{R}^n$, $\nu_1 = \nu_2 = ... = \nu_n$).

The functional and compositional properties of the integral transformation (1) and some of its modifications in spaces $\mathfrak{L}_{\overline{\nu},\overline{2}}$ ($\overline{2} = (2, 2, ..., 2)$, $\overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathrm{R}^n$, $\nu_1 = \nu_2 = ... = \nu_n$) have been studied in the works [1], [2], [5] and [6]. We continue this research. Theory of the considered integral transformation (1) in weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{\mathfrak{r}}}$ is constructed. Mapping properties such as the boundedness, the range, the representation and the inversion of the transform (1) are established. The results presented generalize those obtained in [7, Chapter 4.1] for one-dimensional case.

Refrences

1. Sitnik S. M., Skoromnik O. V., and Shlapakov S. A., *Multi-dimensional generalized integral transform in the weighted spaces of summable functions* // Lobachevskii Journal of Mathematics. 2022. Vol. 43. №6, P. 1170–1178.

2. Sitnik S. M., Skoromnik O. V. *One-dimensional and multi-dimensional integral transforms of Buschman–Erdelyi type with Legendre Functions in kernels* // Transmutation Operators and Applications. Trends in Mathematics. 2020. P. 293–319.

3. Samko S. G., Kilbas A. A., Marichev O. I. *Fractional Integrals and Derivatives: Theory and Applications.* Gordon and Breach Science Publishers, London, 1993.

4. Kilbas A.A., Srivastava H.M., Trujillo J.J. *Theory and applications of fractional differential equations.* Elsevier Science, Amsterdam, 2006.

5. Papkovich M. V., Skoromnik O. V., *Multi-dimensional modified G - transformations and integral transformations with hypergeometric Gauss functions in kernels in weight spaces of summed functions* // Bulletin of the Vitebsk State university. 2022. №1 (114). P. 5–20 [in Russian].

6. Sitnik S. M., Skoromnik O.V., Papkovich M.V., *Multidimensional modified G- and H-transformations and their special cases* // Proceedings of the 10th International Scientific Seminar AMADE-2021, September 13 – 17, 2021, Minsk, Belarus, BSU. Minsk: IVC of the Ministry of Finance. 2022. P. 104 – 116.[in Russian]

7. Kilbas A. A. and Saigo M., *H-Transforms. Theory and Applications.* Chapman and Hall, Boca Raton, 2004.

SPECTRA OF THE ENERGY OPERATOR OF SIX-ELECTRON SYSTEMS IN THE HUBBARD MODEL. SECOND SINGLET STATE

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We consider the energy operator of six-electron systems in the Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system in the second singlet state. Hamiltonian of the system has the form

$$H = A\sum_{m,\gamma} a^{+}_{m,\gamma} a_{m,\gamma} + B \sum_{m,\tau,\gamma} a^{+}_{m,\gamma} a_{m+\tau,\gamma} + U \sum_{m} a^{+}_{m,\uparrow} a_{m,\uparrow} a^{+}_{m,\downarrow} a_{m,\downarrow}. \qquad (1)$$