Our paper is devoted to the study of transform (1) in the weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{\mathfrak{r}}}$ - summable functions $f(\mathbf{x}) = f(x_1, x_2, ..., x_n)$ on $\mathrm{R}^{\mathbf{n}}_{+} = \{\mathbf{x} : \mathbf{x} \in \mathrm{R}^n | \ x_1 > 0, x_2 > 0, ..., x_n > 0\}$, such that

$$\|f\|_{\overline{\nu},\overline{r}} = \{\int\limits_{R^1_+} x_n^{\nu_n \cdot r_n - 1}\{\cdots\{\int\limits_{R^1_+} x_2^{\nu_2 \cdot r_2 - 1}$$

$$\times[\int\limits_{R^1_+} x_1^{\nu_1 \cdot r_1 - 1}|f(x_1, ..., x_n)|^{r_1} dx_1]^{r_2/r_1} dx_2\}^{r_3/r_2} \cdots\}^{r_n/r_{n-1}} dx_n\}^{1/r_n} < \infty$$

( $\overline{r} = (r_1, r_2, ..., r_n) \in \mathrm{R}^n$, $1 \leq \overline{r} < \infty$, $\overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathrm{R}^n$, $\nu_1 = \nu_2 = ... = \nu_n$ ).

The functional and compositional properties of the integral transformation (1) and some of its modifications in spaces $\mathfrak{L}_{\overline{\nu},\overline{2}}$ ( $\overline{2} = (2, 2, ..., 2)$, $\overline{\nu} = (\nu_1, \nu_2, ..., \nu_n) \in \mathrm{R}^n$, $\nu_1 = \nu_2 = ... = \nu_n$ ) have been studied in the works [1], [2], [5] and [6]. We continue this research. Theory of the considered integral transformation (1) in weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{\mathfrak{r}}}$ is constructed. Mapping properties such as the boundedness, the range, the representation and the inversion of the transform (1) are established. The results presented generalize those obtained in [7, Chapter 4.1] for one-dimensional case.

**Refrences**


1. Sitnik S. M., Skoromnik O. V., and Shlapakov S. A., *Multi-dimensional generalized integral transform in the weighted spaces of summable functions* // Lobachevskii Journal of Mathematics. 2022. Vol. 43. №6, P. 1170–1178.

2. Sitnik S. M., Skoromnik O. V. *One-dimensional and multi-dimensional integral transforms of Buschman–Erdelyi type with Legendre Functions in kernels* // Transmutation Operators and Applications. Trends in Mathematics. 2020. P. 293–319.

3. Samko S. G., Kilbas A. A., Marichev O. I. *Fractional Integrals and Derivatives: Theory and Applications.* Gordon and Breach Science Publishers, London, 1993.

4. Kilbas A.A., Srivastava H.M., Trujillo J.J. *Theory and applications of fractional differential equations.* Elsevier Science, Amsterdam, 2006.

5. Papkovich M. V., Skoromnik O. V., *Multi-dimensional modified G - transformations and integral transformations with hypergeometric Gauss functions in kernels in weight spaces of summed functions* // Bulletin of the Vitebsk State university. 2022. №1 (114). P. 5–20 [in Russian].

6. Sitnik S. M., Skoromnik O.V., Papkovich M.V., *Multidimensional modified G- and H-transformations and their special cases* // Proceedings of the 10th International Scientific Seminar AMADE-2021, September 13 – 17, 2021, Minsk, Belarus, BSU. Minsk: IVC of the Ministry of Finance. 2022. P. 104 – 116.[in Russian]

7. Kilbas A. A. and Saigo M., *H-Transforms. Theory and Applications.* Chapman and Hall, Boca Raton, 2004.


## SPECTRA OF THE ENERGY OPERATOR OF SIX-ELECTRON SYSTEMS IN THE HUBBARD MODEL. SECOND SINGLET STATE

**S.M. Tashpulatov**

We consider the energy operator of six-electron systems in the Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system in the second singlet state. Hamiltonian of the system has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma} a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma} a_{m+\tau,\gamma} + U\sum_{m} a^+_{m,\uparrow} a_{m,\uparrow} a^+_{m,\downarrow} a_{m,\downarrow}. \quad (1)$$

Here, $A$ is the electron energy at a lattice site, $B$ is the transfer integral between neighboring sites (we assume that $B>0$ for convenience), $\tau$ which means that summation is taken over the nearest neighbors, $U$ is the parameter of the on-site Coulomb interaction of two electrons, $\gamma$ is the spin index, $\gamma=\uparrow$ or $\gamma=\downarrow$ . and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m\in Z^\nu$.

The Hamiltonian $H$ acts in the antisymmetric complex Foc'k space $(\mathcal{H}_{as},(.)_{\mathcal{H}_{as}})$. Suppose that $\varphi_0$ is the vacuum vector in the space $\mathcal{H}_{as}$. The second singlet state corresponds to the free motion of six electrons over the lattice and their interactions with the basic functions ${}^2s^0_{p,q,r,t,k,n\in Z^\nu}=a^+_{p,\uparrow}a^+_{q,\downarrow}a^+_{r,\uparrow}a^+_{t,\downarrow}a^+_{k,\uparrow}a^+_{n,\downarrow}\varphi_0$. The linear subspace ${}^2\mathcal{H}^0_s$, corresponding the second singlet state is the set of all vectors of the form

$$ {}^2\psi^0_s=\sum_{p,q,r,t,k,n\in Z^\nu} f(p,q,r,t,k,n)^2 s^0_{p,q,r,t,k,n\in Z^\nu}, f\in l_2^{as}, $$

where $l_2^{as}$ is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^6$. We denote by ${}^2H^0_s$ the restriction of operator $H$ to the subspace ${}^2\mathcal{H}^0_s$.

Let $\mathcal{F}:l_2((Z^\nu)^6\rightarrow L_2((T^\nu)^6\equiv {}^2\mathcal{H}^0_s$ be the Fourier transform, where $T^\nu$ is the $\nu-$ dimensional torus endowed with the normalized Lebesgue measure $d\lambda$, i.e. $\lambda(T^\nu)=1$. We set ${}^2\widetilde{H}^0_s=\mathcal{F}\,{}^2H^0_s\,\mathcal{F}^{-1}$.

**Theorem 1.** *The Fourier transform of operator* ${}^2H^0_s$ *is an operator* ${}^2\widetilde{H}^0_s=\mathcal{F}\,{}^2H^0_s\,\mathcal{F}^{-1}$ *acting in the space* $L_2^{as}((T^\nu)^6)$ *be the formula*

$$ {}^2\widetilde{H}^0_s\,{}^2\psi^0_s=h(\lambda,\mu,\gamma,\theta,\eta,\xi)f(\lambda,\mu,\gamma,\theta,\eta,\xi)+ $$

$$ +U\int_{T^\nu}[f(t,\lambda+\mu-t,\gamma,\theta,\eta,\xi)+f(t,\mu,\gamma,\lambda+\theta-t,\eta,\xi)+ $$

$$ +f(t,\mu,\gamma,\theta,\eta,\lambda+\xi-t)+f(\lambda,t,\mu+\gamma-t,\theta,\eta,\xi)+ $$

$$ +f(\lambda,t,\gamma,\theta,\mu+\eta-t,\xi)+f(\lambda,\mu,t,\gamma+\theta-t,\eta,\xi)+ $$

$$ +f(\lambda,\mu,t,\theta,\eta,\gamma+\xi-t)+f(\lambda,\mu,\gamma,t,\theta+\eta-t,\xi)+f(\lambda,\mu,\gamma,\theta,t,\eta+\xi-t)]dt, \quad (2) $$

*where* $h(\lambda,\mu,\gamma,\theta,\eta,\xi)=6A+2B\sum_{i=1}^{\nu}[cos\lambda_i+cos\mu_i+cos\gamma_i+cos\theta_i+cos\eta_i+cos\xi_i]$, *and* $L_2^{as}$ *is the subspace of antisymmetric functions in* $L_2((T^\nu)^6)$.

**Theorem 2.** *a). Let* $\nu=1$, *and* $U<0$, *then the essential spectrum of operator* ${}^2\widetilde{H}^0_s$ *is the union of seven segments:* $\sigma_{ess}({}^2\widetilde{H}^0_s)=[a+c+e,b+d+f]\cup[a+c+z_3,b+d+z_3]\cup[a+e+z_2,b+f+ +z_2]\cup[a+z_2+z_3,b+z_2+z_3]\cup[c+e+z_1,d+f+z_1]\cup[c+z_1+z_3,d+z_1+z_3]\cup[e+z_1+z_2,f+z_1+z_2]$, *and discrete spectrum of operator* ${}^2\widetilde{H}^0_s$ *is consists of a unique eigenvalue:* $\sigma_{disc}({}^2\widetilde{H}^0_s)=\{z_1+ +z_2+z_3\}$, *what lies to the below than the left edge of the essential spectrum of operator* ${}^2\widetilde{H}^0_s$. *Here, and hereafter* $a=2A-4B\cos\frac{\Lambda_1}{2}$, $b=2A+4B\cos\frac{\Lambda_1}{2}$, $c=2A-4B\cos\frac{\Lambda_2}{2}$, $d=2A+ +4B\cos\frac{\Lambda_2}{2}$, $e=2A-4B\cos\frac{\Lambda_3}{2}$, $f=2A+4B\cos\frac{\Lambda_3}{2}$, $z_1=2A-\sqrt{9U^2+16B^2cos^2\frac{\Lambda_1}{2}}$,

$z_2=2A-\sqrt{9U^2+16B^2cos^2\frac{\Lambda_2}{2}}$,

$z_3=2A-\sqrt{9U^2+16B^2cos^2\frac{\Lambda_3}{2}}$.

*b). Let* $\nu=1$, *and* $U>0$, *then the essential spectrum of operator* ${}^2\widetilde{H}^0_s$ *is the union of seven segment's:* $\sigma_{ess}({}^2\widetilde{H}^0_s)=[a+c+e,b+d+f]\cup[a+c+\widetilde{z}_3,b+d+\widetilde{z}_3]\cup[a+e+\widetilde{z}_2,b+f+\widetilde{z}_2]\cup \cup[a+\widetilde{z}_2+\widetilde{z}_3,b+\widetilde{z}_2+\widetilde{z}_3]\cup[c+e+\widetilde{z}_1,d+f+\widetilde{z}_1]\cup[c+\widetilde{z}_1+\widetilde{z}_3,d+\widetilde{z}_1+\widetilde{z}_3]\cup[e+\widetilde{z}_1+\widetilde{z}_2,f+\widetilde{z}_1+\widetilde{z}_2]$, *and discrete spectrum of operator* ${}^2\widetilde{H}^0_s$ *is consists of a unique eigenvalue:* $\sigma_{disc}({}^2\widetilde{H}^0_s)=\{\widetilde{z}_1+ +\widetilde{z}_2+\widetilde{z}_3\}$, *what lies to the above than the right edge of the essential spectrum of operator*

${}^2\widetilde{H}^0_s$. *Here* $\widetilde{z}_1 = 2A + \sqrt{9U^2 + 16B^2 cos^2\frac{\Lambda_1}{2}}$, $\widetilde{z}_2 = 2A + \sqrt{9U^2 + 16B^2 cos^2\frac{\Lambda_2}{2}}$, $\widetilde{z}_3 = 2A + \sqrt{9U^2 + 16B^2 cos^2\frac{\Lambda_3}{2}}$.

We consider the case $\nu = 3$, and $\Lambda_1 = (\Lambda_1^0, \Lambda_1^0, \Lambda_1^0), \Lambda_2 = (\Lambda_2^0, \Lambda_2^0, \Lambda_2^0), \Lambda_3 = (\Lambda_3^0, \Lambda_3^0, \Lambda_3^0)$. We let $W$ denote the Watson integral: [1] $W = \frac{1}{\pi^3}\int_0^\pi\int_0^\pi\int_0^\pi \frac{3dxdydz}{3-cosx-cosy-cosz} \approx 1,516$. Becase the measure $\lambda$ is normalized, $J = \int_{-\pi}^\pi\int_{-\pi}^\pi\int_{-\pi}^\pi \frac{dxdydz}{3-cosx-cosy-cosz} = \frac{W}{3}$.

**Theorem 3.** *A). If* $U < 0$, *and* $U < -\frac{4Bcos\frac{\Lambda_1^0}{2}}{W}$, *and* $cos\frac{\Lambda_1^0}{2} > cos\frac{\Lambda_2^0}{2} > cos\frac{\Lambda_3^0}{2}$, *or* $cos\frac{\Lambda_1^0}{2} > cos\frac{\Lambda_3^0}{2} > cos\frac{\Lambda_2^0}{2}$, *or* $U < 0$, $U < -\frac{4Bcos\frac{\Lambda_2^0}{2}}{W}$, *and* $cos\frac{\Lambda_2^0}{2} > cos\frac{\Lambda_1^0}{2} > cos\frac{\Lambda_3^0}{2}$, *or* $cos\frac{\Lambda_2^0}{2} > cos\frac{\Lambda_3^0}{2} > cos\frac{\Lambda_1^0}{2}$, *or* $U < 0$, $U < -\frac{4Bcos\frac{\Lambda_3^0}{2}}{W}$, *and* $cos\frac{\Lambda_3^0}{2} > cos\frac{\Lambda_2^0}{2} > cos\frac{\Lambda_3^0}{2}$, *and* $cos\frac{\Lambda_3^0}{2} > cos\frac{\Lambda_1^0}{2} > cos\frac{\Lambda_3^0}{2}$, *then the essential spectrum of operator* ${}^2\widetilde{H}^0_s$ *consists of the union of seven segments:* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+c_1+z_3, b_1+d_1+z_3]\cup[a_1+e_1+z_2, b_1+f_1+z_2]\cup$ $[a_1+z_2+z_3, b_1+z_2+z_3]\cup[c_1+e_1+z_1, d_1+f_1+z_1]\cup[c_1+z_1+z_3, d_1+z_1+z_3]\cup$ $[e_1+z_1+z_2, f_1+z_1+z_2]$, *and the discrete spectrum of operator* ${}^2\widetilde{H}^0_s$ *consists of unique eigenvalue:* $\sigma_{disc}({}^2\widetilde{H}^0_s) = \{z_1+z_2+z_3\}$. *Here and hereafter* $a_1 = 2A - 12B\cos\frac{\Lambda_1^0}{2}$, $b_1 = 2A + 12B\cos\frac{\Lambda_1^0}{2}$, $c_1 = 2A - 12B\cos\frac{\Lambda_2^0}{2}$, $d_1 = 2A + 12B\cos\frac{\Lambda_2^0}{2}$, $e_1 = 2A - 12B\cos\frac{\Lambda_3^0}{2}$, $f_1 = 2A + 12B\cos\frac{\Lambda_3^0}{2}$, *and* $z_1$, $z_2$, *and* $z_3$ *are the same concrete numbers.*

*B). If* $U < 0$, $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_2^0}{2}}{W}$, *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_3^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_3^0}{2}}{W}$, *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_2^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_1^0}{2}}{W}$, *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_3^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_3^0}{2}}{W}$, *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_1^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_2^0}{2}}{W}$, *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_1^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_1^0}{2}}{W}$, *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_2^0}{2}$, *then the essential spectrum of operator* ${}^2\widetilde{H}^0_s$ *consists of the union of four segments:* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+c_1+z_3, b_1+d_1+z_3]\cup[a_1+e_1+z_2, b_1+f_1+z_2]\cup[a_1+z_2+z_3, b_1+z_2+z_3]$, *or* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+c_1+z_3, b_1+d_1+z_3]\cup[c_1+e_1+z_1, d_1+f_1+z_1]\cup[c_1+z_1+z_3, d_1+z_1+z_3]$, *or* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+e_1+z_2, b_1+f_1+z_2]\cup[c_1+e_1+z_1, d_1+f_1+z_1]\cup[e_1+z_1+z_2, f_1+z_1+z_2]$, *and discrete spectrum of operator* ${}^2\widetilde{H}^0_s$ *is empty set:* $\sigma_{disc}({}^2\widetilde{H}^0_s) = \emptyset$.

*C). If* $U < 0$, $-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_3^0}{2}}{W}$, *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_3^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_2^0}{2}}{W}$, *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_2^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_3^0}{2}}{W}$, *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_3^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_1^0}{2}}{W}$, *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_1^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_1^0}{2}}{W}$, *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_1^0}{2}$, *or* $U < 0$, $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \le U < -\frac{4B\cos\frac{\Lambda_2^0}{2}}{W}$, *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_2^0}{2}$, *then the essential spectrum of operator* ${}^2\widetilde{H}^0_s$ *consists of the union of two segments:* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+c_1+z_3, b_1+d_1+z_3]$, *or* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[a_1+e_1+z_2, b_1+f_1+z_2]$, *or* $\sigma_{ess}({}^2\widetilde{H}^0_s) = [a_1+c_1+e_1, b_1+d_1+f_1]\cup[c_1+e_1+z_1, d_1+f_1+z_1]$, *and the discrete spectrum of operator* ${}^2\widetilde{H}^0_s$ *is empty set:* $\sigma_{disc}({}^2\widetilde{H}^0_s) = \emptyset$.

*D). If* $U < 0$, $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \le U < 0$, *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_3^0}{2}$, *or* $U < 0$, $-$

$-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \leq U < 0,$ *and* $\cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_2^0}{2},$ *or* $U < 0,$ $-\frac{4B\cos\frac{\Lambda_3^0}{2}}{W} \leq U < 0,$ *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_3^0}{2},$ *or* $U < 0,$ $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \leq U < 0,$ *and* $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_1^0}{2},$ *or* $U < 0,$ $-\frac{4B\cos\frac{\Lambda_1^0}{2}}{W} \leq U < 0,$ *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_1^0}{2},$ *or* $U < 0,$ $-\frac{4B\cos\frac{\Lambda_2^0}{2}}{W} \leq U < 0,$ *and* $\cos\frac{\Lambda_3^0}{2} > \cos\frac{\Lambda_1^0}{2} > \cos\frac{\Lambda_2^0}{2},$ *then the essential spectrum of operator* ${}^2\widetilde{H}_s^0$ *consists of the union of unique segment:* $\sigma_{ess}({}^2\widetilde{H}_s^0) = [a_1 + c_1 + e_1, b_1 + d_1 + f_1],$ *and the discrete spectrum of operator* ${}^2\widetilde{H}_s^0$ *is empty set:* $\sigma_{disc}({}^2\widetilde{H}_s^0) = \emptyset$.

**Refrences**

1. Valkov V. V., and Ovchinnikov S. G., and Petrakovskii O. P. *The Excitation Spectra of two-magnon systems in easy-axis quasidimensional Ferromagnets.* Sov. Phys. Solid State. 1986. Vol. 30. PP. 3044-3047.