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GEODESICS OF RIEMANNIAN METRICS RELATED TO THE NAVIE–STOKES EQUATIONS AND THEIR APPLICATIONS

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1.The $14D$ Riemann -metric in local coordinates $\vec{x}=[x,y,z,t,\eta,\rho,m,u,v,w,p,\xi,\chi,n]$

$$ds^2 = 2dx\,du + 2dy\,dv + 2dz\,dw + (-Uu - Vv - Ww)\,dt^2 + 2dt\,dp +$$

$$+Ad\eta^2 + 2d\eta\,d\xi + Bd\rho^2 + 2d\rho\,d\chi + Cdm^2 + 2dm\,dn,$$

where

$$A = (-UW + \mu\,U_x)\,w + (-UV + \mu\,U_y)\,v + \left(-U^2 - P + \mu\,U_x\right)u - Up,$$

$$B = (\mu\,V_z - VW)w + (-V^2 - P + \mu\,V_y)v + (-UV + \mu\,V_x)\,u - Vp,$$

$$C = (\mu\,W_z - (W^2 - P)w + (\mu\,W_y - VW)v, (-UW + \mu\,W_x)u - Wp,$$

(U,V,W) , P – the components of velocity and pressure of liquid, has the Ricci-tensor of the form $R_{44} = U_x + V_y + W_z = 0$, $R_{55} = 0$, $R_{66} = 0$, $R_{77} = 0$ on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{V} + \left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} - \mu\Delta\vec{V} + \vec{\nabla}P = 0, \quad \vec{\nabla}\cdot\vec{V} = 0.$$

It belongs to the class of Riemann extensions of affinnely-connected spaces, are equipped by partially-projective metrics due the conditions to part of coordinates

$$\ddot{\eta} = 0, \quad \ddot{\rho} = 0, \quad \ddot{m}, \quad \ddot{\xi} = 0, \quad \ddot{\chi} = 0, \quad \ddot{n} = 0$$

and the remaining geodesic equations have the form

$$\ddot{x}^k + \Gamma^k_{ij}\dot{x}^i\dot{x}^j = 0, \quad \ddot{\Psi}_k + T^i_k\Psi_i = 0,$$

where $x = (x,y,z,t)$ -the Euler coordinates, $\Psi_k = (u,v,w,p)$ -the Lagrange coordinates.

2. The four-dimensional metric

$$ds^2 = (2\,za_3(x,y) - 2\,ta_4(x,y))\,dx^2 + 2\,(2\,za_2(x,y) - 2\,ta_3(x,y))\,dxdy + 2\,dxdz +$$

$$+(2\,za_1(x,y) - 2\,ta_2(x,y))\,dy^2 + 2\,dydt,$$

associated with the second order ODE's

$$\frac{d^2y}{dx^2}+a_1(x,y)\left(\frac{dy}{dx}\right)^3+3a_2(x,y)\left(\frac{dy}{dx}\right)^2+3a_3(x,y)\frac{dy}{dx}+a_4(x,y)=0$$

with arbitrary coefficients $a_i(x,y)$ and the metric

$$ds^2=y^2{d_x}^2+2\,\left(l(x,t)y^2-1/2\right)d_xd_t+2\,d_yd_t+\left((l(x,t))^2\,y^2-2\,\left(\frac{\partial}{\partial x}l(x,t)\right)y+l(x,t)\right){d_t}^2$$

which Rcci-flat on solutions the KdV-equation

$$\frac{\partial}{\partial t}l(x,t)-3\,l(x,t)\frac{\partial}{\partial x}l(x,t)+\frac{\partial^3}{\partial x^3}l(x,t)=0$$

together with the six-dimensional metric in local coordinates x,y,t,p,q,s

$$ds^2=4\,p\left(\frac{\partial}{\partial x}f(x,y,t)\right)dxdt+2\,dxdp+4\,p\left(\frac{\partial}{\partial y}f(x,y,t)\right)dydt+2\,dydq+2\,dtds+$$

$$+\left(-2\,pf(x,y,t)\frac{\partial}{\partial x}f(x,y,t)-2\,p\frac{\partial^3}{\partial x^3}f(x,y,t)-2\,\mu\,q\frac{\partial}{\partial y}f(x,y,t)+2\,s\frac{\partial}{\partial x}f(x,y,t)\right)dt^2$$

which is the Ricci-flat $R_{ik}=0$ on solutions of the KP-equation

$$\left(\frac{\partial}{\partial x}f(x,y,t)\right)^2+f(x,y,t)\frac{\partial^2}{\partial x^2}f(x,y,t)+\frac{\partial^4}{\partial x^4}f(x,y,t)+\frac{\partial^2}{\partial t\partial x}f(x,y,t)+\mu\,\frac{\partial^2}{\partial y^2}f(x,y,t)=0$$

may be of used to construct the examples of flows liquids, described by the NS-system of equations.

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