GENERALIZATION OF THE ABRAMS-STROGATTI MODEL OF LANGUAGE DYNAMICS TO THE CASE OF SEVERAL LANGUAGES

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Mathematical modeling is actively used to study language dynamics; many works are devoted to its research [1-7]. The study of language dynamics and the prediction of its results are very important due to the development of digital and telecommunication technologies, the global Internet [8]. The central problem is to find the trend when one language displaces the others, i.e. becomes the dominant, because by this it influences all spheres of social life.

The purpose of this work is to construct and study a generalized mathematical model of Abrams-Strogatti to describe the dynamics of several coexisting languages.

The following model is cosidered:

$$\dot{x_i} = \left(\sum_{j\neq i}^{N} x_j\right) cs_i x_i^{\alpha} - x_i c\left(\sum_{j\neq i}^{N} s_j x_j^{\alpha}\right), \quad \sum_{i=1}^{N} x_i = 1, \quad \sum_{i=1}^{N} s_i = 1, \quad \alpha > 1. \tag{1}$$

The phase variables $x_i, i = \overline{1, N}$, are the fraction of speakers of different languages. The parameter s_i is the prestige of the i-th language, the constant α is the volatility that is detrmined by the Abrams-Strogatti hypotheses [9]. The coefficient c is some constant. The space of states for (1) is the standard simplex [10].

The analytical research is based on the mathematical approach to study systems of differential equations on a standard simplex [10-14], which was used to model the transmission of genetically unfixed information. The language competitiveness function was obtained [15,16]:

$$J_i = cs_i x_i^{\alpha-1}(0).$$

The dominant language is the language for which the function of competitiveness has the gratest value. Competitiveness depends on the initial distribution of speakers in languages.

In this work, ten-year statistics has been analyzed (of the) on the use of the eight most popular languages on the Internet: English, Russian, German, Spanish, Chinese, Japanese, French and Turkish [8]. Based on this, the parameters of the model were identified. It has been shown that the English language displaces other languages from the global Internet over time, under persisting conditions.

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METHODS FOR SOLVING ALGEBRAIC EQUATIONS IN THE THEORY OF LINEAR DIFFERENTIAL EQUATIONS

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Consider a linear homogeneous differential equation with constant coefficients of degree n [1]

$$L(w) = 0, \tag{1}$$

where

$$L() = a_n \frac{d^n}{dz^n} + a_{n-1}\frac{d^{n-1}}{dz^{n-1}} + ... + a_1 \frac{d}{dz} + a_0, \quad a_n \neq 0, \quad a_i \in \mathbb{R}.$$

We will look for the solution of equation (1) in the form

$$w = Ae^{xz},$$

after substituting it into equation (1), we obtain an algebraic equation of degree n [2-4]:

$$y = f(x) = \sum_{i=0}^{n} a_i x^i = 0, \quad a_n \neq 0, \quad a_i \in \mathbb{R}, \tag{2}$$

which on the plane Oxy defines a certain graph of the function, and its solution is to find the intersection points of the graph of the function with the axis Ox or to find the zeros of the function.

We introduce the substitution of variables $x = \varphi(t)$, which, when substituted into equation (2), transforms our graph of the function $y = f(\varphi(t)) = g(t)$. Moving to a new two-dimensional space with coordinates t, y, we will look for the solution of equation (2) there.

We will choose the replacement of variables so that the equation in the new variables has a polynomial form

$$y = g(t) = \sum_{i=0}^{k} b_i t^i = 0, \quad b_k \neq 0. \tag{3}$$