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Soliton-Induced Electric Currents in Plasma

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Abstract—This is a theoretical study of the nonequilibrium motion of charged particles in an electric field of solitons. We show that the self-consistent electric field of ion-acoustic and electron-acoustic solitons is characterized by one-way transfer of charged particles at a distance of several Debye radii. The dependence of relevant local currents on the amplitude of solitons is determined. We consider the practically important case of a moving cascade consisting of many solitons and show that the induced currents have a significant constant component. The kinetic energy acquired by charged particles in the soliton field is calculated. The temporal resolution required for the recording of soliton-induced currents is estimated. The calculations presented here can be used to interpret the results of experiments conducted to study solitons in the space plasma.

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An analysis of the broadband electrostatic noise recorded by the Geotail satellite in the Earth's magnetosphere is presented in [1]. This noise was shown to be associated with the motion of groups of solitons. Later, solitons were found by *Cluster* [2], *Polar* [3], *Fast* [4], *S3 Trio* [5], *Viking* [6] satellites in the auroral zone and other regions of near-Earth space. Often, solitons are recorded in the presence of charged particle beams. For example, the auroral magnetosphere contains inward and outward current regions with a large population of solitons of different types [3]. The coupling between electric fields of solitons with electric fields and currents in the Earth's magnetosphere was noted in [7]. However, the role of solitons in the generation of plasma currents has not been studied exhaustively for several reasons. The main reason is the low temporal resolution of the instruments used for measuring the currents in space plasma [2]. Also, there are theoretical models designed for describing experiments and analyzing the electric fields, potentials, or charged particle concentrations, rather than electric currents.

This study aims to find a coupling between plasma currents and electric fields of solitons. The motion of heavy impurity ions in an electric field of solitons has been analyzed partially in [8]. In the present paper, we propose a generalized theoretical model for analyzing currents arising in electric fields of solitons that is valid for the main plasma components; this model was used to determine the main parameters of currents and show that they have a significant magnitude and can affect many processes occurring in plasma.

1. THEORETICAL MODEL

1.1. Ion-Acoustic Solitons (Model 1). Our basic model is the one-dimensional collisionless MHD model of plasma without a magnetic field and with a hot electron background and cold singly charged ions [9]. The equations of motion and continuity for ions as well as the Boltzmann distribution for electrons and the Poisson equation are

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \varphi}{\partial x}, \quad (1)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_i}{\partial x} = 0, \quad (2)$$

$$n_e = n_0 \exp\left(e\varphi/T_e\right), \quad (3)$$

$$\frac{\partial^2 \varphi}{\partial x^2} = -\frac{e}{\varepsilon_0}\left(n_i - n_e\right). \quad (4)$$

Here, v_i, n_i, and m_i are the velocity, density, and mass of ions, respectively; n_e and T_e are the concentration and temperature of electrons, respectively; n_0 is the unperturbed concentration of electrons and ions, and e is the elementary charge. The potential φ corresponds to the electrostatic field $E = -\partial\varphi/\partial x$. By analogy with [9], we suppose that $T_i = 0$; here, the pressure of ions can be taken as vanishing.

We restrict ourselves to the search of stationary solutions of system (1)–(4). To this end, we introduce a new variable $S = (x - Vt)/\lambda_D$, where V is the steady velocity of a soliton, and $\lambda_D = \sqrt{\varepsilon_0 T_e / e^2 n_0}$ is the Debye radius. In addition, we introduce a normalization for the potential and concentrations of charged particles: $\Phi = e\varphi/T_e$, $N_i = n_i/n_0$, $N_e = n_e/n_0$. Performing a change of variables and a single integration, we write Eq. (2) as

$$N_i(v_i - V) = -V. \quad (5)$$

After a single integration of Eq. (1) and taking into account (5), we obtain an equation for the ion concentration: $M_i^2(1 - N_i^2) + 2\Phi N_i^2 = 0$. The solution of this equation has the form

$$N_i = M_i\Big/\sqrt{M_i^2 - 2\Phi}, \quad (6)$$

where $M_i = V/C_i$ is the Mach number and $C_i = \sqrt{T_e/m_i}$ is the rate of ion sound. In view of the normalization, the electron concentration can be represented as $N_e = \exp(\Phi)$. Then, Eq. (4) takes the form

$$d^2\Phi\Big/dS^2 = \exp(\Phi) - N_i. \quad (7)$$

Solitons are one of the classes of solutions of Eq. (7). In the approximation of small amplitudes ($|\Phi| \ll 1$), these solutions can be written as [9]

$$\Phi(S) = \Phi_0 \operatorname{sech}^2\left(\sqrt{-\frac{A_1}{2}}S\right). \quad (8)$$

Here, $\Phi_0 = -A_1/A_2$ is the soliton amplitude, $A_1 = \frac{1}{2M_i^2} - \frac{1}{2}$, $A_2 = \frac{1}{2M_i^4} - \frac{1}{6}$.

Let us find the electrical current induced by the electric field of a soliton. We restrict ourselves merely to the calculation of the ion current; the calculation of the current of the hot electron component is beyond the scope of this study and will be considered in our future works. The ions are undisturbed because $T_i = 0$. Under the action of the electric field of the soliton, the ions acquire velocity, which can be expressed in the fixed coordinate system with the help of (5) as

$$v_i = V\left(1 - \frac{1}{N_i}\right). \quad (9)$$

Then, the expression for the ion current density takes the form

$$j_i = en_iV\left(1 - \frac{1}{N_i}\right). \quad (10)$$

Normalizing (10) to en_0C_i, we obtain an expression for the normalized density of the ion current:

$$J_i = M_i(N_i - 1). \quad (11)$$

The dependencies of potential Φ, the normalized electric field $\xi = d\Phi/dS$, and the concentrations N_i and N_e on S as well as the dependencies of the normalized rate v_i/C_i and ion current density J_i on X for different times τ are shown in Fig. 1, according to (8), (6), (3), (9) and (11). Hereafter, $X = x/\lambda_D$ and $\tau = t\omega_i$, where $\omega_i = \sqrt{e^2n_0/m_i\varepsilon_0}$ is the ion plasma frequency.

It can be seen from Fig. 1b that the concentration of ions in the field of positive polarity solitons is increased in comparison with the undisturbed state. In view of (9), in this case, the normalized velocity of ions v_i/C_i is positive; i.e., its direction coincides with the soliton motion direction (Fig. 1c). Therefore, the field of an ion-acoustic soliton of positive polarity transfers the ions only in the soliton motion direction. This transfer yields an ion current momentum J_i having a direction that coincides with the direction of v_i (Fig. 1d). Thus, the ion-acoustic solitons actually should generate unipolar pulses of the ion current. The model with two electron populations considered in [10] allows for the existence of ion-acoustic solitons of negative polarity. This case corresponds to a decrease in the normalized ion concentration ($0 < N_i < 1$) and, according to (11), an oppositely directed ion current.

1.2. Electron-Acoustic Solitons (Model 2). The electron-acoustic solitons are analyzed using a three-component model with a hot electron background, a population of cold electrons and fixed singly charged ions [11]. We write the equations of motion and continuity for the cold electron component the Boltzmann distribution for the hot component; we supplement the Poisson equation by assuming that the ion concentration is constant ($n_i = n_0$):

$$\frac{\partial v_{ec}}{\partial t} + v_{ec}\frac{\partial v_{ec}}{\partial x} = \frac{e}{m_e}\frac{\partial \varphi}{\partial x}, \quad (12)$$

$$\frac{\partial n_{ec}}{\partial t} + \frac{\partial n_{ec}v_{ec}}{\partial x} = 0, \quad (13)$$

$$n_{eh} = n_{eh,0}\exp(e\varphi/T_{eh}), \quad (14)$$

$$\frac{\partial^2\varphi}{\partial x^2} = -\frac{e}{\varepsilon_0}(n_0 - n_{eh} - n_{ec}). \quad (15)$$

Here, m_e is the electron mass, the subscripts *ec* and *eh* refer to cold and hot electronic populations, respectively, and $T_{ec} = 0$. The unperturbed electron concentrations $n_{eh,0}$ and $n_{ec,0}$ are related through the quasi-neutrality principle: $n_0 - n_{eh,0} - n_{ec,0} = 0$. Following the reasoning of subsection 1.1, we can reduce system (12)–(15) to the Poisson equation as

$$d^2\Phi\Big/dS^2 = \delta\exp(\Phi) + (1-\delta)N_{ec} - 1, \quad (15)$$

where

$$N_{ec} = M_e\Big/\sqrt{M_e^2 + 2\Phi\frac{\delta}{1-\delta}}. \quad (16)$$

Here, $\Phi = e\varphi/T_{eh}$, $\delta = n_{eh,0}/n_0$, $M_e = V/C_e$, $C_e = \sqrt{T_{eh}n_{ec,0}/m_en_{eh,0}}$ is the electron-acoustic concentration. The temporal and spatial variables are nor-

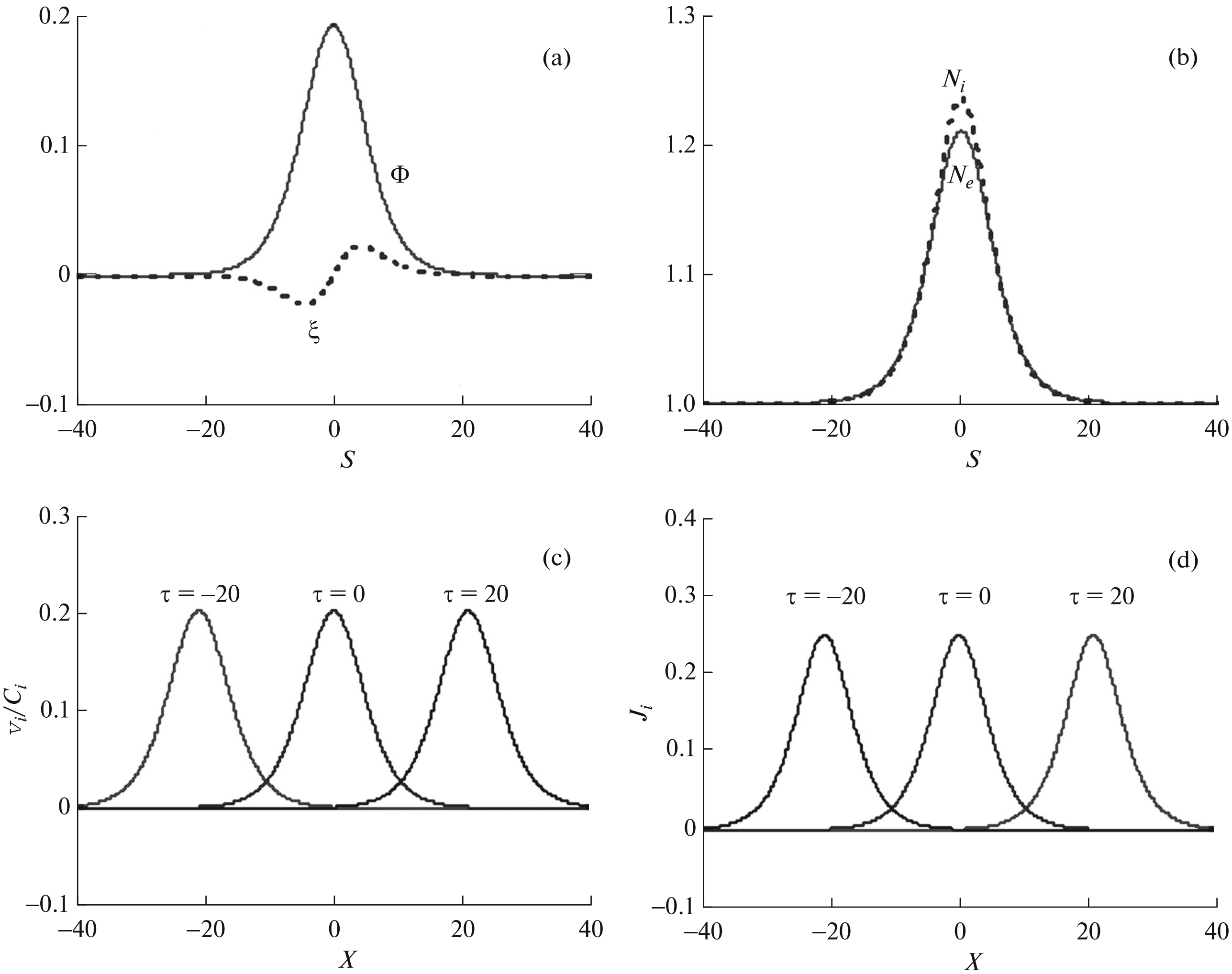

Fig. 1. Characteristics of an ion-acoustic soliton for $M_i = 1.05$: (a) normalized potential Φ and electric field ξ, (b) perturbed concentrations of electrons and ions, (c) normalized ion velocity v_i/C_i for three time instants τ, and (d) ion current density J_i for three time instants τ.

malized to ω_{ec}^{-1} and $\lambda_D = \sqrt{\varepsilon_0 T_{eh}/e^2 n_0}$, where $\omega_{ec} = \sqrt{e^2 n_{ec,0}/m_e \varepsilon_0}$ is the electron plasma frequency for the cold electron population. Taking into account that $A_1 = \frac{\delta}{2M_e^2} - \frac{\delta}{2}$, $A_2 = -\frac{\delta^2}{2M_e^4(1-\delta)} - \frac{\delta}{6}$, soliton solutions (15) can be represented as (8). The electron current caused by the motion of cold electrons in the field of electron-acoustic solitons and normalized $-en_{ec,0}C_e$ can be easily expressed as

$$J_{ec} = M_e(1 - N_{ec}). \tag{17}$$

The calculation the current density of hot electrons is beyond the scope of this study. The ion current is vanishing because the ions are fixed.

The perturbations of the potential, electric field, and concentration of charged particles for the electronic-acoustic soliton as well as the rate and density of the current of cold electrons are shown in Fig. 2. It is clear that the electron-acoustic solitons also generate unipolar current pulses in the plasma; in this case, the negative polarity of the potential corresponds to the negative polarity of pulses of the current of cold electrons.

The resulting equations make it possible to calculate some important parameters of induced currents.

2. CALCULATION OF CURRENT PARAMETERS

We calculate the length of unipolar current pulses induced by solitons, which can be useful for estimating the appropriate time resolution of the experimental equipment mounted on satellites. We define the current pulse length χ as the time during which the condition $J > J_0/\exp(1)$, where J_0 is the pulse amplitude,

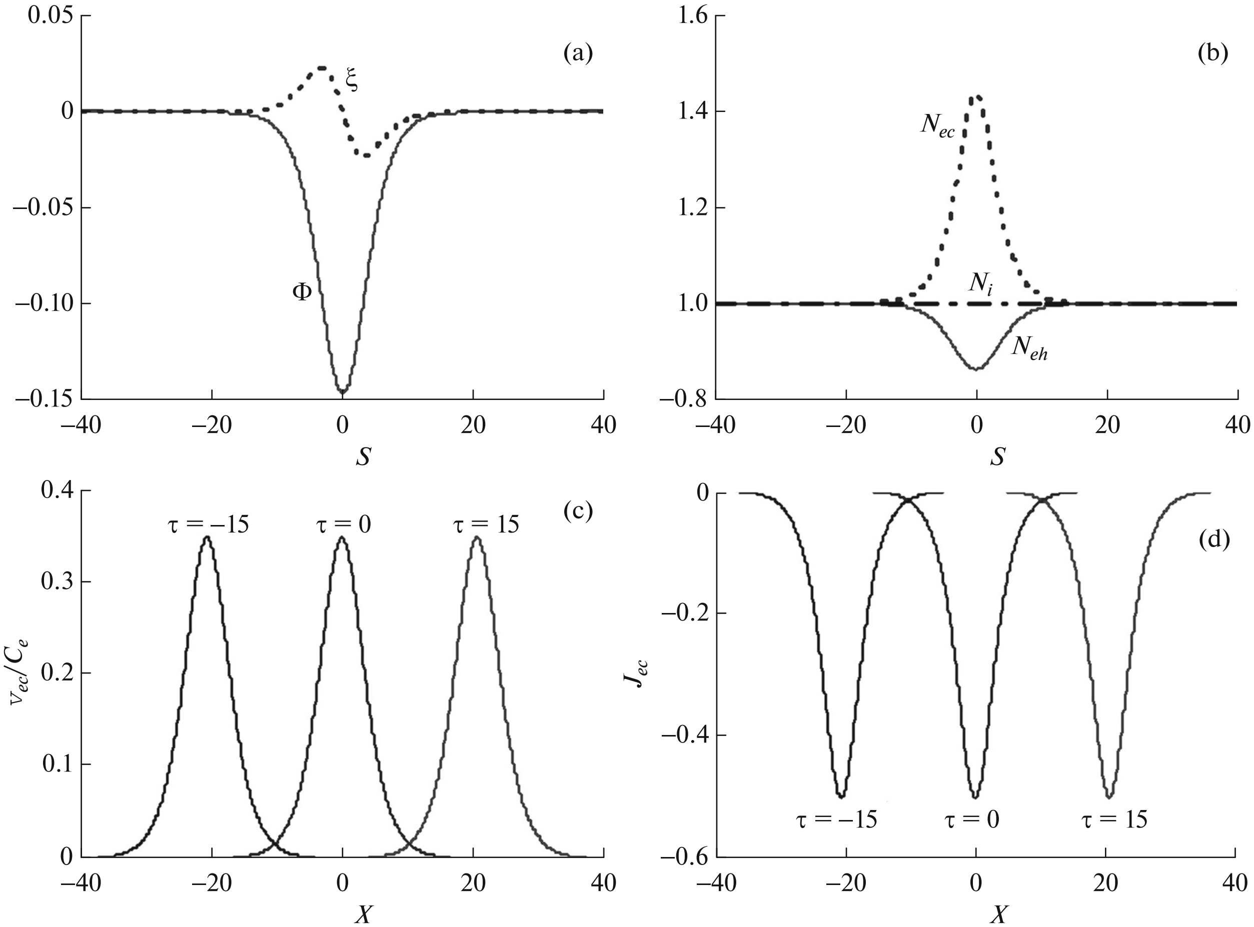

Fig. 2. Characteristics of an electron-acoustic soliton for $M_{ec} = 1.15$, $\delta = 0.7$.

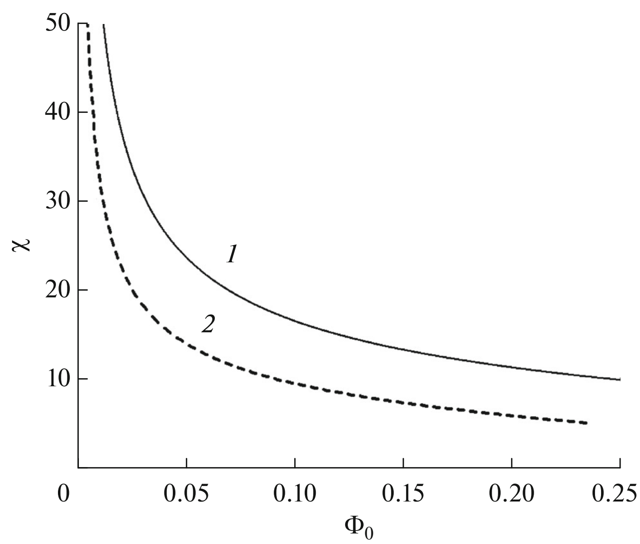

Fig. 3. Dependence of the length of current pulses on the soliton amplitude for the (*1*) first and (*2*) second models at $\delta = 0.7$.

is satisfied. The dependence of the length of ion current pulses on M_i for ion-acoustic solitons can be easily obtained from (11), (6), and (8) as

$$\chi_i(M_i) = \frac{2}{M_i}\sqrt{-\frac{2}{A_1}}\operatorname{arccosh}\left(\frac{B}{M_i}\sqrt{\frac{2\gamma\Phi_0}{B^2-1}}\right), \qquad (18)$$

where $\gamma = 1$, $B = 1 + \frac{1}{\exp(1)}\left(\frac{M_i}{\sqrt{M_i^2 - 2\Phi_0}} - 1\right)$. Using (17), (16), and (8), a similar expression can be obtained also for electron-acoustic solitons by replacing M_i with M_e and assuming that $\gamma = \frac{\delta}{\delta - 1}$, $B = 1 + \frac{1}{\exp(1)}\left(\frac{M_e}{\sqrt{M_e^2 - 2\gamma\Phi_0}} - 1\right)$. The dependence $\chi(\Phi_0)$ can be represented parametrically or graphically (Fig. 3). It can be seen that χ decreases with increasing amplitude of the solitons, which is caused by a decrease in the width of solitons. Also, it follows from Fig. 3 that $\chi \sim 10$ at $|\Phi_0| = 0.2$. Therefore, the experi-

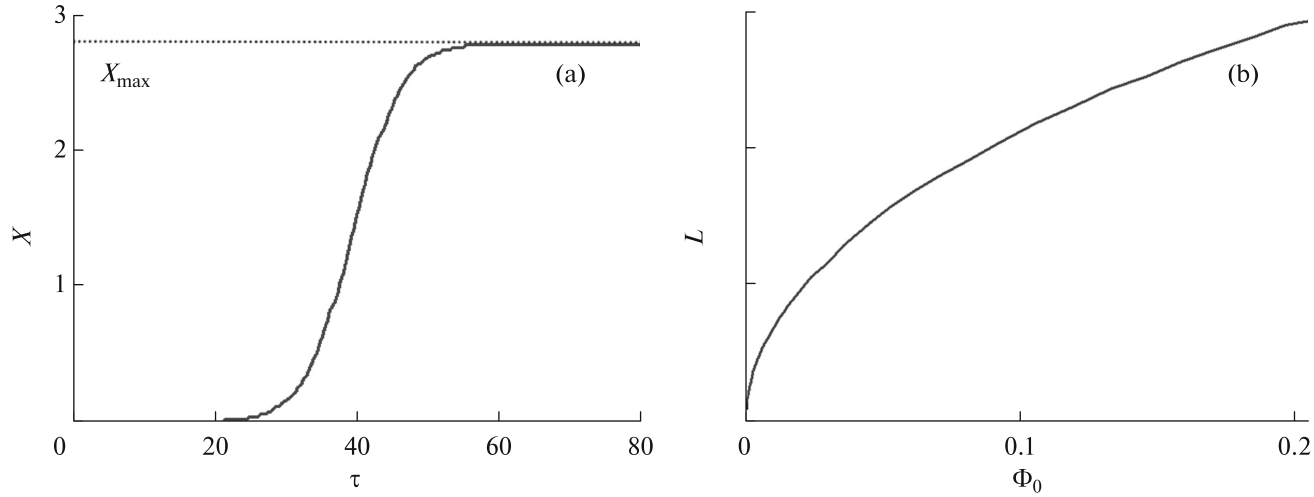

Fig. 4. Solution of Eq. (21) at $M_i = 1.05$: (a) ion trajectory and (b) dependence of the distance of ion transfer by the electric field on the amplitude of ion-acoustic soliton.

mental recording of current pulses of ion- and electron-acoustic solitons requires a temporal resolution of orders ω_i^{-1} and ω_{ec}^{-1}, respectively. Otherwise, the integral value of the current is measured, which can be erroneously attributed to nonexistent beams of charged particles.

On determining the velocities of particles in the electric field of solitons, one can estimate another important and experimentally determined parameter—the kinetic energy of particles W in comparison to their thermal energy. For the first model, we have $W = m_i v_{imax}^2 / 2T_i$. Here, v_{imax} is the maximum value of $|v_i|$. Assuming that $v_{i\,max}/C_i = 0.2$ (see Fig. 1c), we obtain $W = 0.02/\sigma_i$, where $\sigma_i = T_i/T_e$. Normally, $\sigma_i \ll 1$ for space plasma. Let us assume that $\sigma_i = 0.01$; then, $W = 2$. In the second model, a similar estimate can be obtained for the cold electron population. Thus, the kinetic energy of particles in current pulses induced by solitons can be of the order of magnitude of the thermal energy of the corresponding population.

To calculate the motion trajectory and the distance of transfer of individual charged particles in the electric field of solitons, we use the single-particle approximation. We assume that a given charged particle is affected only by the electric field of the soliton because its width significantly exceeds λ_D (Fig. 1a) and, therefore, the Debye screening processes can be disregarded (earlier, we have disregarded the pressure). In this case, the one-dimensional equation of motion of a charged particle of mass m can be written as

$$m\frac{d^2x}{dt^2} = \pm eE. \tag{19}$$

Let us consider the first model in detail. Since $S = X - M_i\tau$, we reduce (19) to the form

$$\frac{d^2X}{d\tau^2} = \frac{d\Phi(X,\tau)}{dX}. \tag{20}$$

The expression for $\Phi(X,\tau)$ can be obtained by substituting $S = X - M_i\tau$ into the right-hand side of (8). Let us assume that the initial ion velocity is zero in (20) due to the supposition $T_i = 0$. The initial ion position is taken to be zero. We place the center of the soliton with amplitude $\Phi_0 = 0.2$ at a point with the coordinate $X = -40$ to ensure that the soliton does not affect the analyzed ion at the initial time. The solution of (20), which describes the ion trajectory, is shown in Fig. 4a. It can be seen that the interaction with leads to the fact that ion position in the space changes. In this case, the ion velocity remains equal to zero before and after the interaction. The distance of transfer is defined as follows: $L = X_{max} - X_0$, where X_0 and X_{max} are the initial and final positions of the ion, respectively. The dependence of $L(\Phi_0)$ obtained from the numerical solution of (20) for solitons of different amplitudes is shown in Fig. 4b. It follows from this figure that one soliton with amplitude $\Phi_0 > 0.05$ transfers ions over a distance of several λ_D. Thus, large groups of solitons, which are often observed in the cosmic plasma [2, 3], can significantly affect the transfer processes.

These results can be generalized to the second model.

4. DISCUSSION OF RESULTS

The results obtained above allow us to draw an important conclusion: ion- and electron-acoustic solitons can transfer charged particles in space. The currents arising here have the form of unipolar pulses (Figs. 1d and 2d). The length of current pulses is tens of ω_i^{-1} and ω_{ec}^{-1} for the first and second models, respectively (Fig. 3). If the time resolution of the measuring equipment is insufficient, the pulse structure of currents cannot be recorded.

In the space plasma, solitons are often recorded as large groups containing tens of gradually moving solitary waves [2, 3]. In this case, the total current induced by a group of solitons can be significant. Let us estimate the current induced by a group of Z closely spaced ion-acoustic solitons with amplitude $\Phi_0 = 0.2$. According to (18), the length of the current pulse is $\chi_0 \approx 10$ at $\Phi_0 = 0.2$. We assume that the solitons cross some fixed point with the periodicity $\tau_0 = 5\chi_0$. This problem qualitatively describes the experimental situation [3]. Figure 4b indicates that a soliton with amplitude $\Phi_0 = 0.2$ transfers the ions over the distance $L_0 \approx 3$. In this case, Z solitons transfer the ions over the distance ZL_0 during the time $\tau = Z\tau_0$. Since the motion involves all ions, we can estimate the average value of the ion current as

$$\overline{j_i} = \frac{ZL_0 n_i e}{Z\tau_0}(\lambda_D \omega_i).$$

The multiplier in the brackets takes into account the normalization of L_0 and τ_0. For the parameters mentioned above, we obtain the value $\overline{j_i} \approx 0.06 e n_0 C_i$ or $\overline{J_i} \approx 0.06$ in the normalized form. For comparison, the same current can be caused by a beam of ions with the concentration $n = 0.06 n_0$ that move with the near-sonic velocity $v = C_i$.

Let us consider the question of the absence of soliton decay at the generation of currents. According to Fig. 4a, the initial and final positions of a charged particle changes when interacting with solitons, while the initial and final velocities of that particle are the same (Fig. 1c). Since the self-consistent motion involves a large number of particles (some of which are accelerated and some are decelerated), the total energy balance is conserved and the soliton does not decay in the absence of collisions. Consequently, the solitons do not transmit energy to charged particles but exchange it with them by participating in adiabatic motion. Thus, it can be supposed that the transfer of charged particles is inherent to solitons with any amplitude and is a fundamental property of solitons.

The electric currents in plasma are normally conditioned by beams of charged particles. It is known that solitons can be amplified in the presence of beams [12, 13]. The results we obtained indicate that solitons themselves can contribute to the recorded electric current, which has the distinctive feature of being pulsed. Thus, one can suppose that there is a nonlinear mechanism of current transformation in plasma, when the beam of charged particles in plasma excites solitons, which in turn induce a current. In this case, the current character undergoes a significant change: the constant current of the beam can be transformed into a pulsating current of solitons (or their superposition).

CONCLUSIONS

Using a MHD model of plasma, we have derived equations to calculate the electric currents induced by ion- and electron-acoustic solitons. We have shown that the solitons can transfer charged particles in space and determined the distance of this transfer. The soliton-induced currents have the form of unipolar pulses. We have estimated the time resolution of the measuring equipment needed to record these pulses. It has been shown that the average value of the ion current caused by the motion of an ensemble of solitons is comparable by its magnitude with the current generated by ion beams. We assume that there is new nonlinear mechanism of the transformation of currents in plasma.

In this paper, we present the calculated parameters of the motion of cold plasma populations in the electric field of solitons. The motion of hot particle populations will be analyzed in future studies.

REFERENCES

1. Matsumoto, H., Kojima, H., Miyatake, T., et al., Electrostatic solitary waves (ESW) in the magnetotail: BEN wave forms observed by Geotail, *Geophys. Res. Lett.*, 1994, vol. 21, pp. 2915–2918.
2. Pickett, J.S., Kahler, S.W., Chen, L.-J., et al., Solitary waves observed in the auroral zone: The cluster multi-spacecraft perspective, *Nonlinear Processes Geophys.*, 2004, vol. 11, pp. 183–196.
3. Bounds, S., Pfaff, R., Knowlton, S., et al., Solitary structures associated with ion and electron beams near 1 Re altitude, *J. Geophys. Res.*, 1999, vol. 104, pp. 28709–28717.
4. McFadden, J.P., Carlson, C.W., Ergun, R.E., et al., FAST observations of ion solitary waves, *J. Geophys. Res.*, 2003, vol. 108, no. A4, p. 8018. doi 10.1029/2002JA009485
5. Temerin, M., Cerny, K., Lotko, W., and Mozer, F.S., Observations of double layers and solitary waves in the auroral plasma, *Phys. Rev. Lett.*, 1982, vol. 48, pp. 1175–1179.
6. Bostrom, R., Gustafsson, G., Hollback, B., et al., Characteristics of solitary waves and weak double layers in the magnetospheric plasma, *Phys. Rev. Lett.*, 1988, vol. 61, pp. 82–85.
7. Galperin, Yu.I. and Volosevich, A.V., Nonlinear electrostatic waves and moving localized structures in the

outer plasmasphere and auroral magnetosphere, *Cosmic Res.*, 2000, vol. 38, no. 5, pp. 514–525.

8. Trukhachev, F.M., Theoretical model of charged particle motion in the electric field of soliton-like structures on the example of hydrogen plasma with Argon impurity ions, *Vesti Nats. Akad. Nauk Belarusi, Ser. Fiz.–Mat. Nauk*, 2005, no. 5, pp. 105–107.
9. Sagdeev, R.Z., Collective processes and shock waves in dilute plasma, in *Voprosy teorii plazmy* (Problems in Plasma Theory), Leontovich, M.A., Ed., Moscow: Atomizdat, 1964, vol. 4, pp. 20–80.
10. Ghosh, S.S., Ghosh, K.K., and Sekar Lyender, A.N., Large Mach number ion acoustic rarefactive solitary waves for a two electron temperature warm ion plasma, *J. Phys. Plasmas*, 1996, vol. 3, pp. 3939–3945.
11. Watanabe, K. and Taniuti, T., Electron-acoustic mode in a plasma of two-temperature electrons, *J. Phys. Soc. Jpn.*, 1977, vol. 43, pp. 1819–1820.
12. Okutsu, E., Nakamura, M., Nakamura, Y., and Itoh, T., Amplification of ion-acoustic solitons by an ion beam, *Plasma Phys.*, 1978, vol. 20, pp. 561–568.
13. Abrol, P.S. and Tagare, S.G., Ion-beam generated ion-acoustic solitons in beam plasma system with non-isothermal electrons, *Plasma Phys.*, 1980, vol. 22, pp. 831–841.

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