Load Capacity of Spherical Roller Transmission with **Double-Row Pinion**

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Abstract. This paper provides an in-depth description of the design and operating principle of a spherical roller transmission producing a wide range of reduction ratios in small dimensions. The rotational motion is transmitted from the drive shaft to the driven shaft by a pinion with two coaxial rows of rollers. The pinion performs a spherical motion about a fixed point on the transmission axis with a constant nutation angle. The rollers are in contact with a fixed periodic closed groove formed by two cams mounted in the housing and a groove made on the cam of the driven shaft. Power is transmitted through a number of parallel flows, which increases the load capacity, i.e., the maximum torque in the driven shaft. The main criterion of reliability is the contact strength of the cam surfaces. Equations for calculating contact stress in the rollergroove contact on the cam of the driven shaft are obtained by use of the Hertz theory. A formula for calculating the maximum torque being transmitted, which is obtained from the kinetostatic equilibrium equations, is given. The analysis of the influence of the main geometric parameters of the transmission on its load capacity is presented.

1. Introduction

Mechanical transmissions with high reduction ratios are used in space and aeronautical systems, drives of positioning and servo systems, shut-off devices, etc. The use of cascade-connected gear sets leads to the greater weight and larger dimensions of the drive, a higher level of noise, increased backlash and, as a result, reduces the reliability of the mechanical system. To obtain high speed reduction with small dimensions, different types of mechanical transmissions are used: worm, wave, planetary gears, etc. Reduction ratios of planetary transmissions with a double-gear pinion can be more than 10,000 [1], but their efficiency is very low. Pin-cycloid gears have a higher efficiency [2], but in both cases the pinion mounted on the drive shaft with eccentricity needs to be balanced. Precession (nutation) transmissions, whose pinion performs a spherical motion about a fixed point [3] (its center of mass), are more balanced. Precession drives with bevel wheels are characterized by low noise and higher load capacity [4].

Spherical roller transmissions (SRT) with a double-row pinion are innovative mechanisms; they are similar in the structure and the principle of operation to precession transmissions. SRTs belong to the class of spherical mechanisms characterized by balance due to the central symmetry, the ability to simultaneously perform several functions (for example, a speed reducer and a spherical joint), as well as by additional kinematic capabilities [5, 6]. SRTs can produce a wide range of reduction ratios, as in the case of planetary transmission with a double-gear pinion. Due to small dimensions, light weight,



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coaxial alignment of shafts and cylindrical (or spherical) housing, they can perform functions of unified mechanical modules and can be integrated into kinematic chains of robots and manipulators. It has been found that the main criterion of reliability of these transmissions is contact strength [7]. It determines the SRT load capacity, the maximum torque being transmitted through the driven shaft. The purpose of this study is to conduct a theoretical assessment of the influence of the main geometric parameters on the SRT load capacity.

2. Design and principle of operation of SRT

Figure 1 shows SRT drive shaft 1 with a section inclined to the transmission axis at an angle Θ . Pinion 2 with the capability to rotate relative to the section is mounted on it. The pinion has two coaxial rows of holes. The rollers with projecting spherical ends are put in these holes with uniform angular spacing. When drive shaft 1 rotates, rollers of the outer row 4 are in contact with periodic groove 7 made in housing 8. Rollers of the inner row 3 are in contact with the groove of end cam 5. Cam 5 is mounted on driven shaft 6 and rotates with it. The groove in the housing is made by two end cams, displaced relative to each other by a quarter period of the groove as shown in figure 2. Both grooves have a rounded profile in the normal section and are produced on cams by means of a spherical mill; the center of the spherical mill, like the center of the spherical end of the rollers, moves along the closed periodic curve located on the spherical surface. The inclination of the pinion axis to the axis of the drive shaft is produced by means of a bushing with a hole, which is inclined at an angle to the axis of the outer surface of the bushing, and two taper washers.



Figure 1. Kinematic diagram of SRT: 1 – drive shaft; 2 – pinion; 3 – rollers of the inner row; 4 – rollers of the outer row; 5 – end cam; 6 – driven shaft; 7 – fixed periodic groove; 8 – housing.



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Figure 2. Key components of SRT prototype: 1 - drive shaft with the double-row pinion; 2 - driven shaft with the end cam; 3 - end cams forming the fixed periodic groove.

The main geometric parameters of SRT are radii of spheres R_2 and R_3 , on which the centers of the spherical ends of the rollers of the inner and outer rows are located, respectively, and the angle of the drive shaft section Θ . The number of periods of the housing groove Z_3 and periodic working surface of the end cam determines the reduction ratio *i*. It can be expressed as:

$$i = Z_2 (Z_3 + 1) (Z_2 - Z_3)^{-1}.$$
(1)

The number of the rollers of the inner row n_{s1} and the outer row n_{s2} is one more than the corresponding number of the periods Z_2 and Z_3 .

The center of mass of the pinion is located on the transmission axis, which improves its balance; dynamic pressures on the pinion bearings are insignificant compared with the forces in engagement. The increase in load capacity is produced by a larger number of rollers simultaneously involved in,

engagement and transmitting load. The rollers are put into the pinion holes by means of bushings made from antifriction material. The rollers are capable of rolling on the groove surface, as in transmissions with intermediate rolling bodies, without sliding. [8–10]. But, in contrast to these transmissions, rollers axes are fixed on a common support, the pinion, which helps reduce noise and avoid jamming.

3. Foundations of SRT contact strength calculations

The condition of contact strength in relation to SRT is obtained by transforming the Hertz equation for the area of contact between the ball and the curved groove [11]:

$$\sigma_{H} = K_{Gs} \xi_{q} N_{m \max}^{\frac{1}{3}} \sum \rho^{\frac{2}{3}} \leq [\sigma_{H}], \qquad (2)$$

where σ_H , $[\sigma_H]$ are the calculated actual and permissible contact stresses, respectively, MPa; N_{mmax} is the maximum value of the normal reaction in the roller-groove contact, N; K_{Gs} is the factor depending on the properties of contacting materials, MPa^{2/3}; ζ_q is the coefficient depending on the factor Ω that in turn depends on differences in the curvature of contacting surfaces. The factor Ω for the contact area of the roller spherical surface and the groove can be expressed as:

$$\Omega = \left(\frac{1}{r_s + \Delta r_s} - \frac{1}{R_2 + r_s}\right) \left(\frac{2}{r_s} - \frac{1}{r_s + \Delta r_s} - \frac{1}{R + r_s}\right)^{-1},$$
(3)

where r_s is the radius of the spherical surface of the roller, mm; Δr_s is the value by which the radius of the groove profile exceeds the radius r_s , mm.

The factor ξ_q can be determined from tables [12] taking into account data interpolation. To automate the calculations, the data from these tables were processed, a dependency graph was created and a 7th-degree polynomial was obtained. It is expressed as:

$$\xi_q = 1.02 - 0.83\Omega + 11.25\Omega^2 - 67.47\Omega^3 + 197.58\Omega^4 - 304.13\Omega^5 + 235.21\Omega^6 - 72.22\Omega^7 \,. \tag{4}$$

With the materials of the parts being the same, the contact stresses in the materials of the cams are determined, because the roller surface is, as a rule, harder. The factor K_{Gs} is calculated according to the following expression:

$$K_{Gs} = 0.364 \left(\left(1 - \mu_r^2 \right) E_r^{-2} + \left(1 - \mu_t^2 \right) E_t^{-2} \right)^{-\frac{2}{3}},$$
(5)

where $\mu_{r(t)}$ is the Poisson's ratio, which depends on the properties of the materials of the rollers (*r*) and the groove (*t*); $E_{r(t)}$ is the modulus of elasticity, MPa.

The maximum value of normal reactions will be achieved in the contact area between the inner row rollers and the cam mounted on the driven shaft, since $R_2 < R_3$, and only half of the rollers can be engaged at the same time. To determine the average value of the normal reaction N_{2m} , a mathematical model of the force interaction of the rollers with the cams surfaces was developed by the kinetostatic method. Thus, we obtain the following expression:

$$N_{2m} = \frac{T_2 K_{nsf} \cos \psi}{0.707 K_n K_{p2} \left(R_2 + 0.707 r_s\right) n_{s2} \sin\left(\alpha_{2m} + \psi\right)},\tag{6}$$

where T_2 is the nominal torque on the driven shaft, Nm; K_{nsf} is the factor used to determine mean values of the radii of the circles formed by the portions of the spherical surface cut off by planes perpendicular to the transmission axis; ψ is the friction angle, rad; K_n is the factor describing uneven distribution of load on the rollers; K_{p2} is the ratio of the number of rollers transmitting the load to their total number indicating the number of the rollers transmitting the load to their average inclination of the groove on the cam of the driven shaft, rad.

The friction angle corresponds to the reduced friction coefficient $f: \psi = \operatorname{atan} f$. The average groove inclination is determined on the plane unfolding of the spherical curve when replacing it with a piecewise helical curve (the vertices of the periodic curve are connected by line segments). It is expressed as $\alpha_{2m} = \operatorname{atan}(2Z_2\Theta\pi^{-1})$.

The factors included in the expression (6) can be calculated as follows:

$$K_{Nsf} = 0.25 (1 + \cos \Theta)^{-1} .$$
 (7)

$$K_{p2} = \left(1 - 0.707 r_s \tan \alpha_{m2} \cdot \sin \alpha_{m2} \left(\Theta \cdot R_2\right)^{-1}\right) Z_{s2}, \qquad (8)$$

where Z_{s2} is the factor depending on the type of closure of kinematic pairs formed by the rollers and the groove. With self-closed kinematic pairs, if the groove is formed by two cams, $Z_{s2} = 1.0$. With force-closed kinematic pairs, the rollers are in contact with one cam, and the number of rollers simultaneously transmitting the load is reduced by half. Thus, $Z_{s2} = 0.5$.

Substituting formula (6) into expression (2), and replacing the computed contact stresses σ_H with their permissible values $[\sigma_H]$, we obtain the equation for the torque T_2 , which is the maximum torque that can be transmitted $T_{2\text{max}}$ based on the contact strength criterion:

$$T_{2\max} = S \frac{\left[\sigma_{H}\right]^{5} K_{n} K_{p2} \left(R_{2} + 0,707 r_{s}\right) n_{s2} \sin\left(\alpha_{2m} + \psi\right)}{\xi_{a}^{3} \Sigma \rho^{2} K_{nsf} \cos\psi},$$
(9)

where $S = 1.098 \cdot 10^{12}$ is the factor taking into account the dimensions of the parameters in formula (9).

4. Evaluation of the influence of SRT parameters on the load capacity

The main geometric parameter of SRT which affects the value of the torque being transmitted and its efficiency is the groove inclination α_{2m} . This inclination, in turn, depends on three independent parameters: the radius R_2 , the angle Θ (or the groove amplitude $A_2 = R_2\Theta$) and the number of periods Z_2 related to the reduction ratio *i*.

Let us consider the transmission whose cams and rollers are made from AISI 5115 steel (DIN 16MnCr5) with working surface hardened to 60...62 HRC by carburizing. In this case, the yield strength is $\sigma_T = 930$ MPa, the permissible contact stresses in prolonged operation is $[\sigma_H] = 1150$ MPa. For steel parts $E_r = E_t = 2.1 \cdot 10^5$ MPa, $\mu_r = \mu_t = 0.3$. With the materials of rollers and grooves being the same, $K_{Gs} = 863.6$ MPa^{2/3}. The numbers of periods are $Z_2 = 15$, $Z_3 = 13$, the reduction ratio is i = 105. The radius of the spherical end of the roller is assumed to be equal to the radius of the groove profile in the normal section, i.e., $\Delta r_s = 0$ mm.

The results of calculations with fixed $R_2 = 38$ mm are shown in figure 3. The values of the experimental factor $K_n = 0.9$, which corresponds to parts accuracy class 7 and the reduced friction coefficient f = 0.02, which takes into account the rollers' rolling and sliding [13], were assumed to be constant. Thus, the study of the influence of geometric parameters of SRT on its load capacity at the specified radial dimensions and reduction ratio has been performed.

Theoretically, an increase in r_s should cause an increase in the torque being transmitted, because the radii of curvature of the contacting surfaces and the contact areas increase as well, and contact stresses are replaced by bearing stresses. There is an optimal value of the radius r_s based on the criterion of maximum load capacity. There are some factors that have the most significant influence. When the groove inclination α_{2m} decreases, which is equivalent to a decrease in Θ and Z_2 and an increase in R_2 , the radii r_s , which produce the extrema $T_{2\max}$, increase, and, as a result, the values of the torques increase. The influence of the angle Θ (and, respectively, α_{2m}) with a stepwise change in r_s can be observed in figure 4. It should be noted that there are optimal values of α_{2m} (and Θ) based on the maximum efficiency criterion. Small groove inclinations cause an increase in wedging forces; at certain values this can lead to SRT's self-braking and jamming.

Another important factor affecting the SRT load capacity is the ratio K_{p2} . When the radius r_s increases, the working sections of the groove-roller contacting areas are reduced.

There is also a limit on the maximum value of the radius based on the smooth operation criterion. The maximum limit of the radius can be calculated as follows:

$$r_{s\max} = \left(1 - \frac{1}{Z_{s2}n_{s2}}\right) \frac{R_2\Theta}{0.707\tan\alpha_{2m}\sin\alpha_{2m}} \,. \tag{10}$$

When this value is exceeded, the average number of the rollers simultaneously transmitting the load becomes less than one, which leads to failure of the engagement smoothness.



of the roller spherical end.

Another restriction is related to arranging rollers in the SRT pinion by analogy with the planet gear arrangement in the planetary gear. This geometric condition of avoiding interference on the roller surfaces can be expressed by the maximum value of the radius r_{sgeo} corresponding to the specified R_2 and n_{s2} : $r_{sgeo} = R_2 \sin(\pi/n_{s2})$.



Figure 4. SRT load capacity dependence on the angle Θ .

Table 1 presents the maximum torque values $T_{2\text{max}}$ for each Θ and respective values r_s , as well as the restrictions imposed on the radius value r_s . In addition to the efficiency, the shear stresses acting in

the roller cross section perpendicular to its axis and the bearing stresses arising in the contact area between the roller and the pinion hole must be taken into account.

Θ (rad)	$T_{2\max}$ (Nm)	r_s (mm)	r_{smax} (mm)	r _{sgeo} (mm)
0.075	81.993	6.8	8.457	7.413
0.085	73.120	6.3	7.814	7.413
0.095	66.743	5.9	7.329	7.413
0.105	61.994	5.6	6.995	7.413
0.115	58.358	5.3	6.661	7.413
0.125	55.506	5.1	6.424	7.413
0.135	52.231	5.0	6.232	7.413

Table 1. Influence of SRT parameters on the load capacity.

5. Conclusions

A mathematical model has been developed, which makes it possible to calculate the main geometric parameters of SRT with specified radial dimensions and reduction ratio and achieve the maximum load capacity taking into account computed contact stresses. To increase the load capacity, the radius r_s of the spherical ends of the rollers should be increased and the groove inclination α_{2m} should be reduced. When the radius r_s increases, the maximum torque being transmitted increases as well, but the range of the angles Θ being used is narrowed down. Finally, the value of α_{2m} must be adopted taking into consideration its influence on the SRT efficiency. An increase in the radius R_2 , with the other parameters being constant, leads to an increase in the radial dimensions of SRT, a decrease in α_{2m} and, consequently, an increase in its load capacity. When estimating the influence of the reduction ratio *i*, we must take into account the fact that it depends on the difference in the numbers of the groove periods $Z_2 - Z_3$. In any case, an increase in the number of the periods Z_2 results in an increase α_{2m} and a decreased SRT load capacity determined according to the contact strength criterion.

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