Matter transport as fundamental property of solitons. Generalization of the Stokes drift mechanism to strongly nonlinear systems.

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A soliton is defined as a nonlinear solitary wave that propagates in environmen t with constant speed, shape and amplitude due to a balance of nonlinearity and dispersion. Solitary wave was first described by J. S. Russell in 1884. The mathematical theory of solitary waves was created by Korteweg and De Vries almost half a century later. The new theory caused a significant stir in the scientific community. Indeed, as follows from the famous equation of Korteweg-De Vries (KdV), the soliton profile has an asymptotic $f(x) \sim \text{sech}^2 x$ for small amplitudes. This means that the soliton remains nonlinear for arbitrarily small amplitudes and does not turn into a linear wave. Further studies have shown that solitons are general phenomenon of nature that describes the properties of nonlinear ion-acoustic waves, magneto-acoustic waves, electric currents in nonlinear transmission lines, and much more. A large number of scientific papers have been devoted to the study of soliton properties, but the physics of nonlinear waves and solitons is far from complete.

The goal of this work is to study the ability of solitary waves to transport matter [1]. On the one side, a soliton is a wave. As expected, material waves do not carry matter (they transfer momentum and energy). It is known that this statement is true only for linear waves of infinitely small amplitude. However, for finite amplitudes waves (even harmonic ones), nonlinear effects lead to the emergence of non-zero drift of matter. This phenomenon was predicted in 1847 by Stokes and was named after him (Stokes drift). As is known, the drift speed for a harmonic wave of small but finite amplitude is proportional to its square. Subsequently, the phenomenon of Stokes drift was repeatedly observed in practice for waves on the water surface, acoustic waves, etc. In the Stokes drift situations, the particle motion represented by a superposition of drift and oscillatory motions. Decrease in the wave amplitude leads to a linearization of the wave process and subsequent rapid (quadratic) decrease in the drift component. In this way, for small amplitude harmonic waves, this nonlinear phenomenon is usually neglected.

In the case of solitons, the nonlinearity cannot be neglected. It is shown theoretically that the unidirectional transport of matter (over a finite distance in the direction of soliton motion) is a fundamental property of KdV solitons. It is also shown that the matter transport cannot be neglected as the wave amplitude decreases (in contrast to the Stokes drift), because the magnitude of the transport decreases in proportion to the square root of the soliton amplitude. Due to the universality of the KdV equation, we expect generalization of our results to a wide range of nonlinear problems.

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