THE INFLUENCE OF MASS AND RIGIDITY PARAMETERS ON TORSIONAL VIBRATIONS OF SHAFTS

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Abstract: In this paper the characteristic series method is proposed to solve the boundary value problem of torsion al vibrations of an elastic, variable rigidity, shafts with attached discs. The general form of characteristic equation by means Cauchy function in power series form are obtained. Influence of an attached discs and variable rigidity on the basic frequency are investigated for different boundary conditions. Investigation of influence of variable torsion al rigidity of elastic shafts with attached discs their natural frequency.

1. Introduction

Many rotor machines are based on elastic shafts with variable torsion al rigidity and carrying discrete masses in discs form. The exact solution of a boundary value problem of torsion al vibration of these shafts in a closed form is possible only in a few simple cases. Of course this problems can be solved by means of numerical methods, for example finite elements method. However lack of the exact solution does not allow us to estimate the accuracy of approximate analytical and numerical solutions.

In particular cases likes coal mill when it is necessary to study the dynamics of these units, it is appropriate to use the dual mass model, which is the most simple and yet sufficiently accurately describes the phenomenon of torsion al vibration. In cases where the moment of inertia of the clutch discs cannot be neglected three discs in model rotor of mill will be more accurate. The dependence, derived below provides engineers with a fool enabling design of three – disk units. It permits a choice of appropriate characteristics of elasticity to prevent the occurrence of resonant vibration during starting and stopping of such machines.

2. Definition of the problem

The following boundary problem is considered of model presented on fig.1:

$$L[y] - \sum_{i=1}^{v} \alpha_i y(x_i)\delta(x - x_i) = 0, \quad (a \le x \le b), \tag{2.1}$$

$$(f(x)y' - e_1 y)_{x=a} = 0, \qquad (f(x)y' + e_2 y)_{x=b} = 0, \tag{2.2}$$

where

$$L[y] \equiv (f(x)y')' + \omega^2 \mu(x) y, \qquad a < x_1 < x_2 < \cdots < x_v < b, \tag{2.3}$$

$\alpha_i = J_i \lambda^2$, $\lambda = \sqrt{-1\omega}, \omega$ – frequency parameter, $\delta(x) -$ Dirac's function, $J_i(i = \overline{1,v})$ – central momentum of inertia for symmetrical discs, concentrated in point $x_i, e_j(j = 1,2)$ – parameters of boundary conditions, y(x) – amplitude of torsion angle, function's f(x) and μ(x) characterizing torsion rigidity and mass of shaft are considered as integrated and satisfying such conditions:

$$f(x) > 0, \mu(x) \geq 0, \forall\ x \in [a, b].$$

3. The general form of characteristic equation

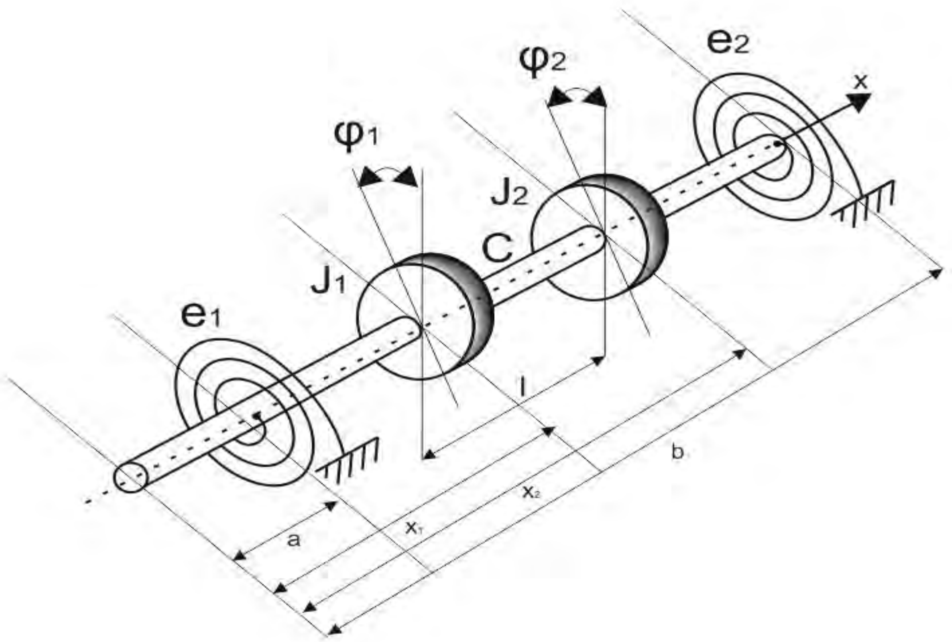

Fig. 1. Model of shafts

Frequency equation of problem for model from fig.1 is obtained in the following form:

$$\left(e_1 e_2 Q + e_1 f(b) Q' - e_2 f(a) \dot{Q} - f(a) f(b) \dot{Q'}\right)\Big|_{\substack{x = b \\ \alpha = a}}$$

(3.1)

This function Q = Q(x, α), is determinate by the formula:

$$Q_{vs} = +\sum_{i=1}^{v} \alpha_i \Psi_i \phi_i + \sum_{i=1}^{v-1} \sum_{j>i}^{v} \alpha_i \alpha_j \Psi_i K_{ji} \phi_{xj} + \sum_{i=1}^{v-2} \sum_{j>i}^{v-1} \sum_{k>j}^{v} \alpha_i \alpha_j \alpha_k \Psi_i K_{ji} \phi_{xk} +$$
$$+ \cdots + \alpha_1 \alpha_2 \ldots \alpha_v \Psi_1 K_{21} K_{32} \ldots K_{v,v-1} \phi_{xv}. \qquad (3.2)$$

In which $\Psi \equiv K(x, \alpha)$, parameter $\alpha = a, \Psi(x_i)$ is marked as Ψ_i,:

$$K(x_j, x_i) = K_{ji},\ \phi(x, x_i) = \phi_{xi},\ Q' = \frac{\partial Q}{\partial x},\quad \dot{Q} = \frac{\partial Q}{\partial \alpha},\ \dot{Q'} = \frac{\partial^2 Q}{\partial x \partial \alpha}, \qquad (3.3)$$

$K(x, \alpha)$ is the influence function of the equation L[y] = 0, i.e. their solution satisfying the following conditions:

$$k(\alpha, \alpha) = 0,\ K'(\alpha, \alpha) = \frac{1}{f(\alpha)(a \leq \alpha \leq b)}, \qquad (3.4)$$

is Heaviside’s function.