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Research Paper

Calculation of Geometric Parameters of an Eccentric Transmission with Intermediate Rolling Elements

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Abstract. In this study, the geometric analysis of an eccentric transmission with intermediate rolling elements is performed. The outer surface of the disk is mounted on the drive shaft. The rolling elements interact with the outer surface of the disk, the inner cam profile of the central wheel, and the surfaces of the radial grooves of the driven shaft with eccentricity. The transmission has low material consumption. The transmission provides transmission ratios in the range of 6…60 in a single stage. The design does not have an additional mechanism for transmitting rotation from the satellite to the driven shaft. The motion trajectories of the centers of mass of rolling elements are obtained. The trajectory equations are the equations of the center curves of the cam profile of the center wheel. An algorithm for calculating the main geometric parameters of transmissions is developed. Experimental samples of reducers based on eccentric transmission are created.

Keywords: Transmission with intermediate rolling elements, eccentric transmission, ball and roller transmission, ball reducer, center curves of cam profiles.

1. Introduction

The development of modern electromechanical drive systems is aimed at reducing their material consumption and dimensions. Gearless drives with high-torque DC motors and frequency regulation of the rotation speed of electric motor shafts have become widespread. At the same time, the use of energy-efficient mechanical transmissions with a high gear ratio is still relevant. Planetary transmissions with cycloidal-pin engagement, designed according to the *k-h-v* scheme, can provide a difference in the number of teeth of the central wheel and satellite equal to one, which makes it possible to increase the gear ratio in one stage, compared to involute transmissions with the same dimensions [1, 2]. The works of Kudriavtsev [1] and Molyneux [2] form the basis of involute gear planetary transmissions, and the proposed results are assumed to be classical results. These studies provide an opportunity to compare the advantages of different transmission models in comparison with involute gear planetary transmissions, i.e., their advantages compared to classical results.

Studies on the use of mechanisms such as parallel crank mechanisms, Hooke couplings to eliminate the relative rotation of the satellite on the transmission drive shaft continue to this day. Efremenkov et al. [3] conducted an analysis of power friction losses in gear engagement with intermediate rolling elements and a free cage. Lustenkov et al. [4, 5] performed a geometric synthesis and comparative analysis of dynamic characteristics of spherical and eccentric transmissions with double-ring satellites. An eccentric transmission with intermediate rolling elements, in which the constancy of their angular position relative to the axis of the driven shaft is ensured, does not require the use of an additional mechanism. When the main elements of the transmission are mounted axially, wedging forces arise, therefore a scheme with a radial arrangement of shafts may be more logical. In recent years, some studies have been conducted on these issues. Among them, Terada and colleagues [6-8] analyzed the motion behavior of orthogonal output type precession and reciprocating ball reducers and then presented an improved design of orthogonal axis type precession ball reducers. Terada [9] made attempts to

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develop rolling ball gearless reducers. Xu and Yang [10] conducted relative velocity and meshing efficiency analysis for a novel planar ball reducer. Song et al. [11] proposed a new sliding ratio modeling method for oscillatory roller transmission based on the velocity decomposition using the mathematical model and kinematic model. Pabiszczak and Kowal [12] investigated the efficiency of a new eccentric rolling transmission, which is a simplified version of a cycloidal reducer with parallel but non-overlapping shaft axes. Liu et al. [13] applied a spatial locomotive-track coupled dynamics model with an eccentric rotor motor and the detailed mechanical structures of the traction motor and motor bearings were comprehensively studied. Xu and Yan [14] presented a double-stage movable tooth reducer with a cam wave generator, which has a small axial size. Pabiszczak [15] evaluated the performance parameters of a cam-type continuously variable transmission each consisting of rollers connected to swing rods mounted on overrunning clutches. Wang et al. [16] studied accurately predict the nonlinear characteristics of transmission efficiency of cycloid reducers under different operating conditions. Khalilov and Sofiyev [17] studied the dynamic behavior of shaft, couplings and working body of the machine under torsional impact moment. It should be emphasized that the designs of such and similar transmissions were developed and patented in the early twentieth century [18, 19].

In this study, geometric analysis of eccentric transmission with intermediate rolling elements is carried out. Rolling elements interact with the outer surface of the disk mounted eccentrically on the drive shaft, the inner cam profile of the central wheel and the surfaces of the radial grooves of the driven shaft. The transmission has low material consumption, provides transmission ratios in the range of 6...60 in one stage. The design does not have an additional mechanism for transmitting rotation from the satellite to the driven shaft. First, the equations of the center curves of the cam profile of the central wheel, the trajectories of the centers of mass of the rolling elements are obtained. Then, an algorithm is developed to calculate the main geometric parameters of the transmissions. Finally, experimental examples created for reducers based on an eccentric transmission are presented and analyses are performed. One of the most important advantages of this design compared to previous studies is the simplification of the design and the other is the increase in efficiency.

The paper is envisaged to be structured as follows: In Section 2, equations for the center profile of the cam surface of the central wheel are derived. In Section 3, the algorithm for calculating the main geometric parameters of an eccentric transmission is presented. After the numerical analysis and comments in Section 4, conclusions are presented in Section 5.

2. Derivation of Equations for the Center Profile of the Cam Surface of the Central Wheel

Let us consider an eccentric mechanism containing a crank drive shaft presented in Fig. 1. Here (a) is the frame after crank rotation, (b) is the frame in the initial position, 1 denotes the drive shaft with crank and 2 denotes the disk mounted on the crank with eccentricity. We will associate a fixed frame of reference $Ox_0y_0z_0$ with the rack, and a frame $Ox_1y_1z_1$ with the crank. The origins of these frames of reference coincide. At the initial moment of time, the axes also coincide, their position is shown in Fig. 1(b). The geometric parameters of the mechanism in the initial position are assigned the index “0”.

A disk 2 with radius $r = R_m$ is mounted on a crank with eccentricity $OC = A$. A moving frame of reference $Ox_2y_2z_2$ is connected to disk 2, the origin of which coincides with the center of disk C.

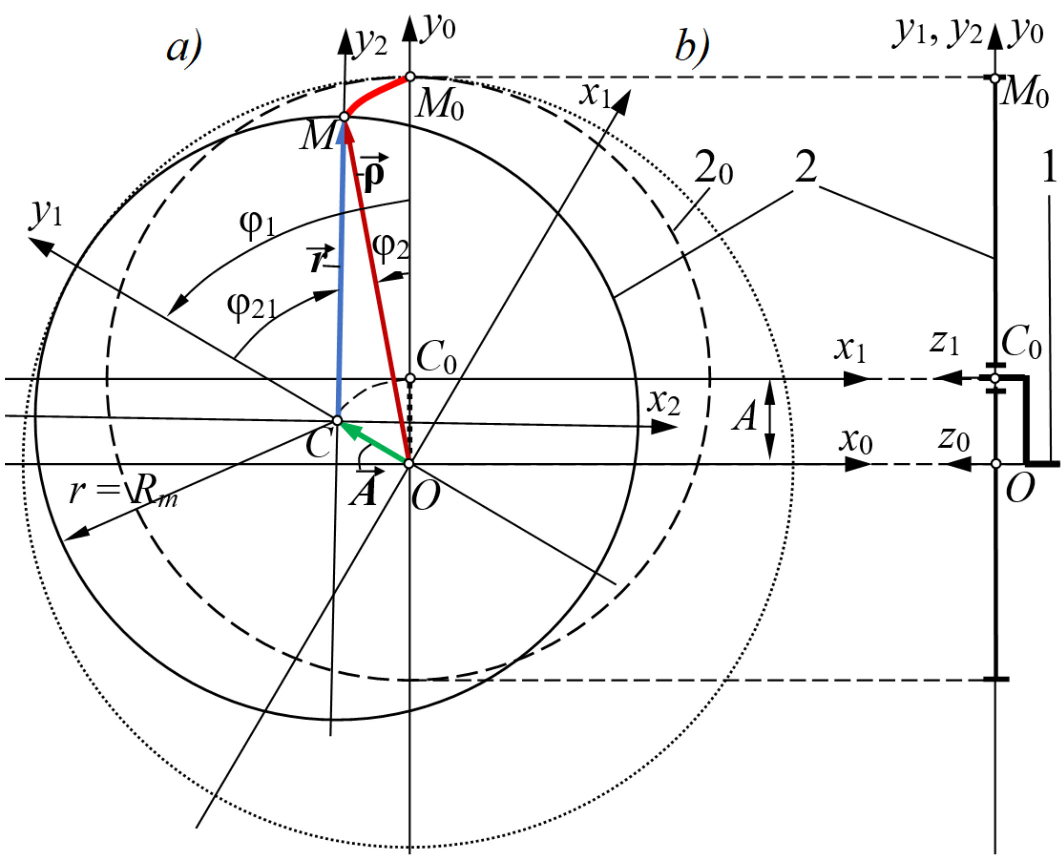

Fig. 1. Eccentric mechanism containing a crank drive shaft.

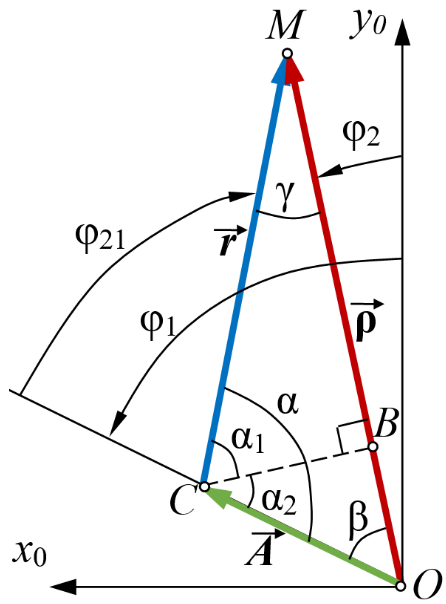

Fig. 2. A vector triangle created to determine the interdependence of angles φ_{21} on φ_1.

Let us consider the rotation of the drive shaft 1 together with the crank and the system $Ox_1y_1z_1$ connected to them counterclockwise by an angle φ_1 (Fig. 1(a)). The disk 2 should rotate in relative motion clockwise by an angle φ_{21} so that the point M in the fixed reference system, which was in the initial position M_0 with coordinates $(0; r + A; 0)$, would occupy a position in which the central angle φ_2, the model of which we are considering. The gear ratio (u) in all mechanical transmissions is determined by the ratio of the rotation frequency (φ_1) of the driving link to the rotation frequency of the driven link, i.e., $\varphi_2 = \varphi_1 / u$. The point M coincides with the center of mass of the rolling element, and the angle φ_2 determines the rotation of the driven shaft of the transmission, which is not shown in the model.

We will apply the matrix method to derive the equations of the trajectories of the centers of mass of the rolling elements. The radius vector determines the position of point M in the moving system and the transition matrices as:

$$r_{M2} = \begin{pmatrix} 0 \\ r \\ 1 \end{pmatrix}, T_{21} = \begin{pmatrix} \cos(-\varphi_{21}) & -\sin(-\varphi_{21}) & 0 \\ \sin(-\varphi_{21}) & \cos(-\varphi_{21}) & A \\ 0 & 0 & 1 \end{pmatrix}, T_{10} = \begin{pmatrix} \cos(\varphi_1) & -\sin(\varphi_1) & 0 \\ \sin(\varphi_1) & \cos(\varphi_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

The matrix T_{21} considers that the angle φ_{21} changes clockwise (negative values). After multiplying the matrices, we obtain:

$$r_{M0} = T_{10} \cdot T_{21} \cdot r_{M2} = \begin{pmatrix} r \cdot \sin(\varphi_{21} - \varphi_1) - A \cdot \sin(\varphi_1) \\ r \cdot \cos(\varphi_{21} - \varphi_1) + A \cdot \cos(\varphi_1) \\ 1 \end{pmatrix} \tag{2}$$

It is necessary to establish the relationship between the angles φ_{21} and φ_1. To do this, consider the vector equality:

$$\vec{A} + \vec{r} = \vec{\rho}. \tag{3}$$

To determine the interdependence of angles φ_{21} on φ_1, let's consider the vector triangle in Fig. 2.

We will choose the positive direction of the abscissa axis to the right, coinciding with the positive direction of the angle count (counterclockwise). To determine the magnitude of the vector ρ, we will project equality (3) onto the axis coinciding with the direction of this vector:

$$A \cdot \cos(\beta) + \sqrt{r^2 - CB^2} = \rho. \tag{4}$$

Considering that $CB = A\sin(\beta)$, and the angle β is equal to:

$$\beta = \varphi_1 - \varphi_2 = \varphi_1 \cdot \left(1 - \frac{1}{u}\right). \tag{5}$$

We obtain expression (4) in the form:

$$\rho = \sqrt{r^2 - A^2 \cdot \sin^2\left(\varphi_1 \cdot \left(1 - \frac{1}{u}\right)\right)} + A \cdot \cos\left(\varphi_1 \cdot \left(1 - \frac{1}{u}\right)\right). \tag{6}$$

Dependencies for other angles:

$$\alpha = \pi - \varphi_{21}, \tag{7}$$

$$\gamma = \varphi_{21} - \varphi_1 + \varphi_2 = \varphi_{21} - \varphi_1 \cdot \left(1 - \frac{1}{u}\right), \tag{8}$$

$$\alpha_1 = \frac{\pi}{2} - \gamma,\ \alpha_2 = \frac{\pi}{2} - \beta. \tag{9}$$

Let us project the vector equality (3) onto the axis perpendicular to the vector ρ (coinciding with the segment *CB*). We obtain:

$$r \cdot \cos(\alpha_1) = A \cdot \cos(\alpha_2). \tag{10}$$

After the appropriate substitutions and transformations (considering the equality $\cos(\pi/2 - x) = \sin(x)$), we obtain the desired dependence of the angle φ_{21} on φ_1:

$$\varphi_{21}(\varphi_1) = \arcsin\left[\frac{A}{r} \cdot \sin\left(\varphi_1 \cdot \left(1 - \frac{1}{u}\right)\right)\right] + \left(\varphi_1 \cdot \left(1 - \frac{1}{u}\right)\right). \tag{11}$$

Using the relation (11), the nonlinear distributions of the angle φ_{21} depending on the angle φ_1 are numerically calculated for five different cases and plotted as curves 1, 2, 3, 4 and 5 in Fig. 3. The values of the *A/r* and *u* which used in the numerical calculations are presented on the Fig. 3. As can be seen from Fig. 3, the nonlinearity increases with the increase of the ratio *A/r* in all five cases. A change in the modulus *u* of the gear ratio changes the angle of inclination of the curves to the abscissa axis, all other things being equal.

In Fig. 4, flat curves are shown which 1 denotes the section of the trajectory of the center of mass of the rolling element during one revolution of the drive shaft, 2 denotes the trajectory of the center of mass of the rolling element according to equations (15) & (16) and (17) & (18), 3 denotes the average circle and 4 denotes the eccentric circle. When the angle $\varphi_1 = 0 \ldots 2\pi$ rad changes, which corresponds to one full revolution of the driving shaft with the eccentric, the point *M* in the fixed frame *Oxyz* of reference will describe part of the curve 1 in Fig. 4. To form a closed trajectory of the center of mass of the rolling element, it is necessary for the driving shaft to make *u* revolutions. In this case, the driven shaft will make one revolution, and the center of mass of the rolling element *M*, moving together with the driven shaft, will return to its original position.

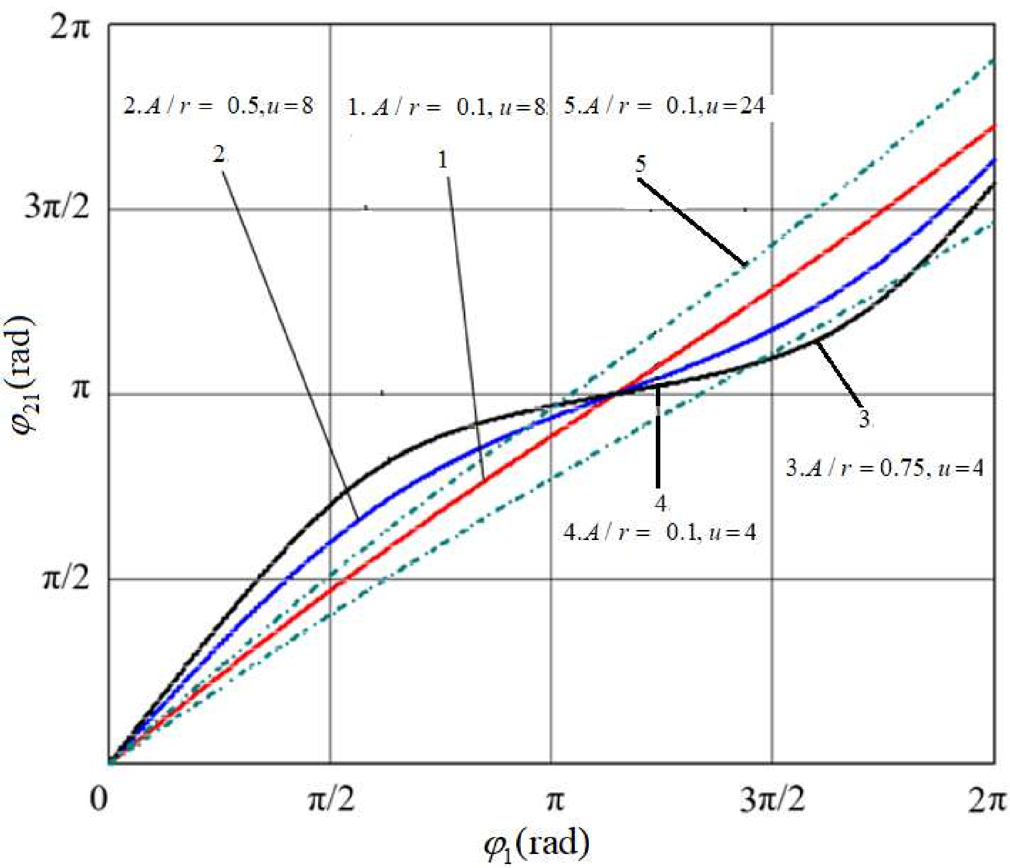

Fig. 3. Nonlinear variation of angle φ_{21} depending on the angle φ_1 with different *A/r* and *u*.

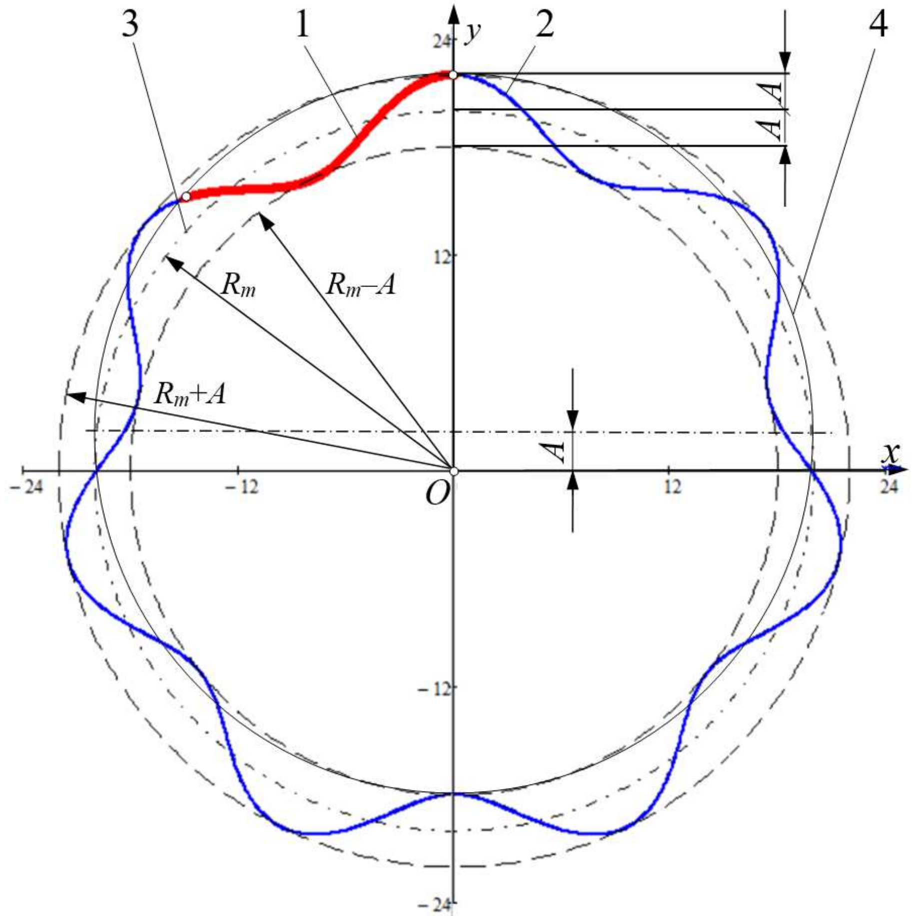

Fig. 4. Flat curves with parameters R_m = 20 mm, A = 2 mm, u = 8.

To construct this trajectory in the upper two elements of the single-column matrix (2), and in expression (11), instead of the angle φ_1, we substitute $\varphi_1 u$. We obtain:

$$\varphi_{21}(\varphi_1) = \arcsin\left(\frac{A}{r}\cdot\sin(\varphi_1\cdot(u-1))\right) + (\varphi_1\cdot(u-1)), \tag{12}$$

$$x_M = r\cdot\sin(\varphi_{21}-\varphi_1\cdot u) - A\cdot\sin(\varphi_1\cdot u), \tag{13}$$

$$y_M = r\cdot\cos(\varphi_{21}-\varphi_1\cdot u) + A\cdot\cos(\varphi_1\cdot u). \tag{14}$$

The closed curve 2 in Fig. 4 constructed using equations (12) to (14) can be classified as periodic. The points moving along the curve perform oscillatory movements in the radial direction relative to the center line (circle 3) with an amplitude equal to A. In the theory of transmissions with intermediate rolling elements in the proposed scheme, the radius r corresponds to the radius of the middle circle R_m, which is easy to prove geometrically. The number of waves (periods) of the curve is the number of teeth Z of the central wheel and is related to the gear ratio by the dependence $Z = u - 1$.

Taking into account the above, equations (12) to (14) can be represented as parametric equations of the curve:

$$x(\varphi) = R_m\cdot\sin\left(\arcsin\left(\frac{A}{r}\cdot\sin(Z\cdot\varphi)\right)-\varphi\right) - A\cdot\sin(\varphi\cdot(Z+1)), \tag{15}$$

$$y(\varphi) = R_m\cdot\cos\left(\arcsin\left(\frac{A}{r}\cdot\sin(Z\cdot\varphi)\right)-\varphi\right) + A\cdot\cos((Z+1)\cdot\varphi). \tag{16}$$

where φ is the central angle, replacing the angle of rotation of the drive shaft φ_1, varying from 0 to 2π rad.

In the monograph [20], the equations of the curve are determined using equation (6): $x(\varphi)=\rho(\varphi)\cdot\sin(\varphi)$, $y(\varphi)=\rho(\varphi)\cdot\cos(\varphi)$. In this case:

$$x(\varphi) = \left(\sqrt{R_m^2 - A^2\cdot\sin^2(Z\cdot\varphi)} + A\cdot\cos(Z\cdot\varphi)\right)\cdot\sin(\varphi), \tag{17}$$

$$y(\varphi)=\left(\sqrt{R_m^2-A^2\cdot\sin^2(Z\cdot\varphi)}+A\cdot\cos(Z\cdot\varphi)\right)\cdot\cos(\varphi). \quad (18)$$

Numerical calculations showed the identity of the curves constructed using equations (15) – (16) and (17) – (18).

To determine the coordinates of the centers of mass of the balls, it is necessary to substitute $\varphi-(2\pi k/n)$ instead of the parameter φ in equations (17) – (18), where k is the rolling element number, changing in the range 0, 1, 2, …, n-1.

3. Algorithm for Calculating the Main Geometric Parameters of an Eccentric Transmission

The profile of the teeth of the central wheel is an equidistant to a closed center curve. The design of the intermediate rolling element transmission implementing the geometric dependencies described above is presented in Fig. 5, such that 1 indicates the drive shaft (eccentric), 2 indicates the driven shaft (separator), 3 indicates the fixed central wheel, and 4 indicates the rolling elements (balls).

Let us consider the algorithm for calculating a transmission with intermediate rolling elements, designed according to the $k-h-v$ scheme.

The initial data are: the transmission ratio module u, the diameter of the middle circle D_m, which determines the radial dimensions of the transmission, the design dimensions δ_r, δ_a, δ_c (Fig. 5) and the diameter of the balls d_s. The number of teeth of the central wheel is determined by the previously given dependence $Z = u - 1$.

The diameter of the eccentric:

$$D_1=D_m-d_s. \quad (19)$$

Width of eccentric (transmission section):

$$b_w=d_s+2\cdot\delta_a. \quad (20)$$

The diameter and width of the eccentric are consistent with the width and diameter of the bearing, which can act as an element in contact with the balls. The bearing mounted on the eccentric has a surface with high hardness. In addition, it provides the mechanism with an additional degree of mobility for self-installation and reducing the overdetermination of the structure. Cage wall thickness:

$$s_2=k_s\cdot d_s. \quad (21)$$

where k_s is the coefficient determining the wall thickness depending on the ball diameter. Based on the separator strength condition, it is recommended to take $k_s=0.5$.

The outer and inner diameters of the separator are respectively:

$$D_{22}=D_m+s_2,\ D_{21}=D_m-s_2. \quad (22)$$

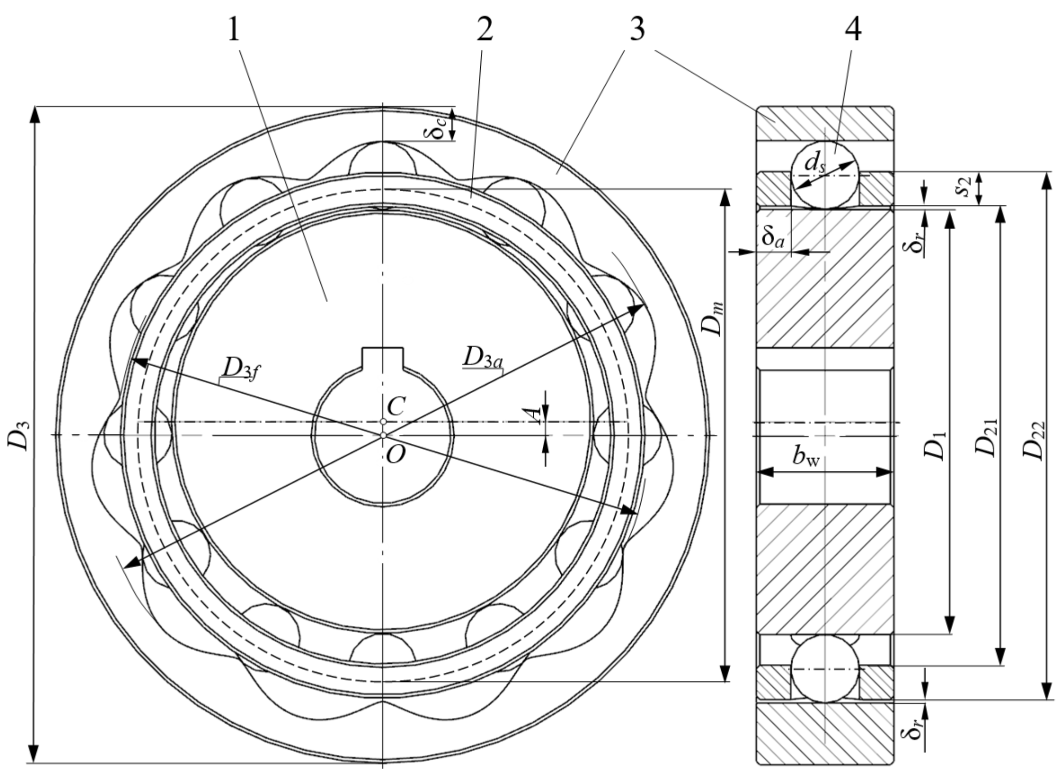

Fig. 5. Design of a transmission with intermediate rolling elements.

We check the condition of the possibility of placing rolling elements (the condition of the proximity of balls). We determine the minimum thickness of the bridge between the holes, measured by the inner diameter of the separator, which should be greater than the permissible value:

$$\delta_{s2} = D_{21} \cdot \sin\left[\frac{\pi}{Z} - \arcsin\left(\frac{d_s}{D_{21}}\right)\right] \geq 0.6 \cdot d_s. \tag{23}$$

Maximum permissible amplitude of the center curve (eccentricity), based on the height of the teeth of the central wheel:

$$A_{\max 1} = r_s - 0{,}5 \cdot s_2 - \delta_r. \tag{24}$$

where r_s denotes the radius of the rolling element, $r_s = 0.5 \cdot d_s$.

Maximum permissible amplitude, based on the constant contact of the balls with the separator:

$$A_{\max 2} = 0.5 \cdot s_2. \tag{25}$$

The minimum is taken for the two values calculated using formulas (24) and (25). If the minimum is the value of A_{max2}, then it must be additionally reduced by 0.5…1 mm when determining A_{max}. Based on the maximum value of the amplitude, its actual value is taken: $A \leq A_{max}$.

The diameter of the tops of the teeth of the central wheel: $D_{3a} = D_m - 2 \cdot A + d_s$. In this case, the following conditions must be met: $D_{3a} \geq D_{22} + 2 \cdot \delta_r$. The diameter of the gullets of the teeth of the central wheel: $D_{3f} = D_m + 2 \cdot A + d_s$. The outer diameter of the central wheel: $D_3 = D_{3f} + 2 \cdot \delta_c$.

The presented algorithm reflects only geometric dependencies between the parameters. The specified parameters D_m and d_s are determined based on the contact strength condition. The ratio A / R_m has an optimal value according to the criterion of maximum efficiency at the established friction coefficients. Rollers can be used as rolling elements, but the misalignment of the axes can cause jamming of the mechanism. To balance and increase the load capacity, eccentric transmissions are made double row, with an eccentricity offset by an angle of π.

4. Results and Discussion

One of the disadvantages of the design of the transmission shown in Fig. 5 is that during operation the rolling elements contact three surfaces simultaneously. The rolling elements can roll without sliding, contacting no more than two surfaces at a time, therefore, increased sliding is observed in the transmission, which causes a decrease in efficiency and wear of the treadmills. Figure 6a shows experimental samples of gear units created on the basis of an eccentric transmission. A special feature is the use of composite rollers instead of balls [21], which increases the efficiency of the gear unit, since each component of the roller has the ability to roll without sliding along the corresponding treadmill. The rotation of the drive shaft with eccentric 2 (Fig. 6a) forces the composite rolling elements 4 (rollers with bearings) to move along a multi-period running track made on the end of the housing disk 3 and along the radial grooves of the driven shaft 1, forcing the latter to rotate at a lower angular velocity than the speed of the eccentric. It is well known that the working path formed on the end of the housing disk and along the radial grooves of the drive shaft is called multi-period running track.

In the transmission under consideration, a shifted circle and a multi-period curve conjugate with it, the equations of which were obtained above, are used to manufacture treadmills. The absence of through grooves increases the strength of the driven shaft. Despite the fact that the housing disk, the driving and driven shafts are installed in the axial direction, wedging forces do not arise due to the design of the composite rollers. Bearings fixed to the ends of the rollers ensure rolling without slipping in all kinematic pairs of the reducer, which increases its efficiency and load capacity. The design of the connecting elements of the reduction sections allows them to be assembled sequentially, using the modular principle (Fig. 6b). The gear ratio of a three-section reduction gear will be equal to 729 with a gear ratio of one section equal to nine.

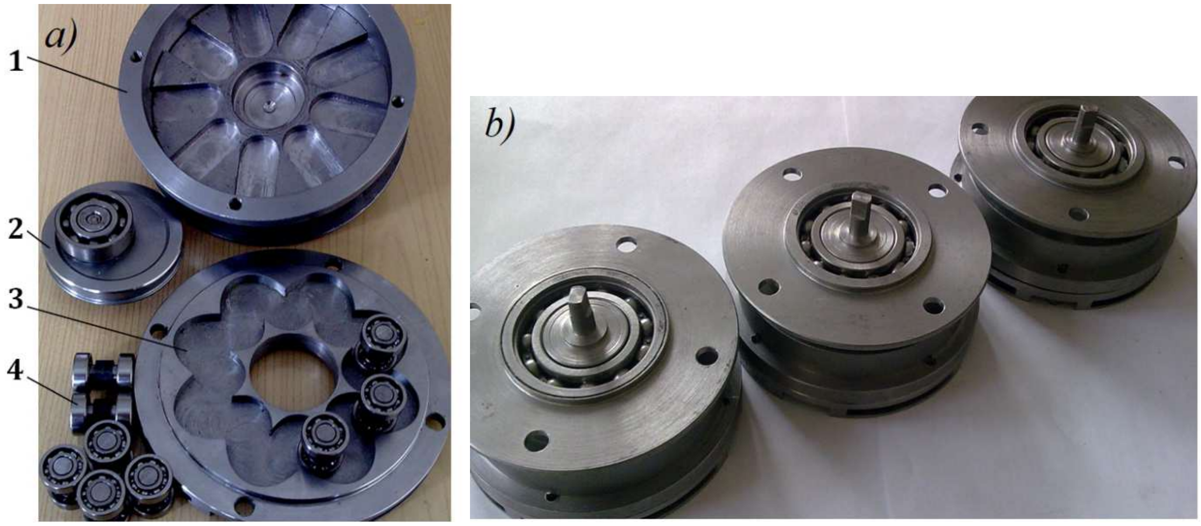

Fig. 6. Experimental samples of reduction units based on eccentric transmission: (a) main transmission elements; (b) three reduction sections assembled.

The obtained equations of the center curves and the dependences of the geometric parameters allow us to perform kinematic and force analyses of the eccentric transmission. In the transmission under consideration, it is advisable to implement gear ratios in the range of 6...60. Smaller values result in the load being transferred by a small number of rolling elements (theoretically, the load is simultaneously transferred by half of the balls from the total number). Larger values increase the radial dimensions of the transmission, or, if specified, reduce the total length of the working sections of the cam profile of the central wheel.

Advantages of the transmission are under consideration, compared with the transmission according to the k-h-v scheme with cycloidal-pin engagement: does not require an additional motion take-off mechanism, this function is performed by the driven shaft; the possibility of some self-adjustment of the balls and compensation for manufacturing and assembly errors. Disadvantages: sliding of the rolling elements relative to the grooves of the driven shaft, since the balls cannot simultaneously roll on three surfaces that do not lie in the same plane; Rolling elements whose axes (centers of mass) are not fixed to a common base cause additional noise and vibration; point contact in ball drives reduces their load capacity. Skew of the roller axes can cause the drive to jam. The proposed design with composite rollers allows for increased efficiency and durability of the eccentric drive.

5. Conclusion

In this study, geometric analysis of eccentric transmission with intermediate rolling elements was carried out. Rolling elements interact with the outer surface of the disk mounted eccentrically on the drive shaft, the inner cam profile of the central wheel and the surfaces of the radial grooves of the driven shaft. The transmission has low material consumption, provides transmission ratios in the range of 6...60 in one stage. The design does not have an additional mechanism for transmitting rotation from the satellite to the driven shaft. First, the equations of the center curves of the cam profile of the central wheel, the trajectories of the centers of mass of the rolling elements are obtained. Then, an algorithm is developed to calculate the main geometric parameters of the transmissions. Finally, experimental examples created for reducers based on an eccentric transmission are presented and analyses are performed. The proposed design with composite rollers allows for increased efficiency and durability of the eccentric drive.

Author Contributions

M.E. Lustenkov played a role in conceptualization, methodology, design, software, validation, formal analysis, visualization, partial manuscript writing, project leadership and administration; A.H. Sofiyev played a role in literature analysis, methodology, software, formal analysis, visualization and partial manuscript writing; I.A. Khalilov played a role in design, manuscript editing, software, validation, interpretation of numerical results and visualization. The manuscript was written thanks to the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request

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