SEPARATION RESULT FOR DISCRETE VOLTERRA EQUATIONS

A. Babiarz, A. Czornik, M. Niezabitowski

In this paper we consider asymptotic properties of solutions of discrete Volterra equations of convolution type. This type of equations has been widely used as a mathematical model in population dynamics [1]. Results on stability and boundedness of solutions of Volterra difference equations may be found in [2, 3]. Very deep and general results about the exact rate of decay of the solution of this type of equations are obtained in [4]. Yet another type of qualitative results about Volterra difference equations such as oscilation, convergency and stability are presented in the papers [5–7]. The main objective of this paper is to present an asymptotic lower bound for the norm of difference of two solutions.

Let us consider the following discrete convolution Volterra equation

$$x(n+1) = x(0) + \sum_{k=0}^{n} a(n-k)A(k)x(k) \quad (n \in \mathbb{N}), \tag{1}$$

where $(A(n))_{n\in\mathbb{N}}$ is a bounded sequence of $d \times d$ matrices

$$\sup_{n\in\mathbb{N}} \|A(n)\| = M < \infty,$$

$(a(n))_{n\in\mathbb{N}}$ is a decreasing sequence of positive numbers satisfying

$$a(n) \leqslant \frac{\overline{M}}{n^{\alpha}}$$

for all $n \in \mathbb{N} \setminus \{0\}$, certain $\alpha \in (0,1)$ and $\overline{M} > 0$.

The next theorem contains the main result of our paper, which shows that the norm of difference of two solutions of equation (1) tends to infinity slower than n^{λ} with certain positive λ.

Theorem. *Let* $\lambda > (1-\alpha)/\alpha$, $x, y \in \mathbb{R}^d$ *and* $x \neq y$. *Then*

$$\limsup_{n\to\infty} n^{\lambda}\|\varphi(n,x) - \varphi(n,y)\| = \infty,$$

where $\varphi(n,x)$ *is the solution of (1) corresponding to the initial condition* $x(0) = x$.

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