ON SUFFICIENT ROBUST CONDITIONS OF FUNCTION SPACE CONTROLLABILITY FOR LINEAR SINGULARLY PERTURBED SYSTEMS WITH MULTIPLE DELAYS ON THE BASIS OF DECOUPLING TRANSFORMATION

O.B. Tsekhan

Consider the singularly perturbed linear time-invariant system with multiple commensurate delays in the slow state variables (SPLTISD) in the operator form:

$$M(\mu)pz(t)=\mathsf{A}(e^{-ph})z(t)+Bu(t),\quad z=\mathrm{col}\,(x,y),\quad x\in\mathbb{R}^{n_1},\quad y\in\mathbb{R}^{n_2},\quad u\in\mathbb{R}^{r},\quad t\geqslant 0,\quad (1)$$

$$z(0)=z_0,\quad z_0\in\mathbb{R}^{n_1+n_2},\quad x(\theta)=\varphi(\theta),\quad \theta\in[-lh,0).\quad (2)$$

Here $0<h$ is a given constant, $p\equiv d/dt$ is a differential operator; e^{-ph} is a delay operator: $e^{-ph}z(t)\equiv z(t-h)$; μ is a small parameter, $\mu\in(0,\mu^0]$, $\mu^0\ll 1$, $M(\mu)=\mathrm{diag}\,\{E_{n_1},\mu E_{n_2}\}$; x is a slow variable, y is a fast variable, u is a control, $u(t)\in U$, U is a set of piecewise continuous for $t\geqslant 0$ vector function;

$$\mathsf{A}(e^{-ph})=\begin{pmatrix}\mathsf{A}_1(e^{-ph}) & A_2\\ \mathsf{A}_3(e^{-ph}) & A_4\end{pmatrix},\quad B=\begin{pmatrix}B_1\\ B_2\end{pmatrix}$$

are matrix-valued operators and matrix, respectively, $\mathsf{A}_\mathsf{i}(e^{-ph})\equiv\sum_{j=0}^{l}A_{ij}e^{-jph}$, $i=1,3$, A_{ij}, $i=1,3$, $j=\overline{0,l}$, A_k, $k=2,4$, B_j, $j=1,2$, are constant matrices of appropriate sizes; $\phi(\theta)$, $\theta\in[-lh,0)$, is a piecewise continuous n_1-vector-function. Assume that $\det A_4\neq 0$.

Definition 1. For a given $\mu\in(0,\mu^0]$ the SPLTISD (1) is said to be complete controllable if for any fixed initial conditions (2) there are the time moment $t_1<+\infty$ and piecewise continuous control $u(t)$, $t\in[0,t_1]$, such that for this control and corresponding solution $z(t,\mu)$, $t\geqslant 0$, of system (1) with the initial conditions (2) the following identities are valid: $z(t,\mu)\equiv 0$, $u(t)\equiv 0$, $t\geqslant t_1$.

Let $\mathbb{R}^n$ be n-dimensional Euclidean space, $L_2^n[t_1-h,t_1]$ – Hilbert space of integrable $n-$ vector-functions on a segment $[t_1-h,t_1]$, $\mathcal{M}_2^n\equiv L_2^n[t_1-h,t_1]\times\mathbb{R}^n$.

Definition 2. For a given $\mu\in(0,\mu^0]$ the SPLTISD (1), (2) is said to be controllable in the space $\mathcal{M}_2^{n_1+n_2}$, if the subspace

$$Z(\mu)=\{z(t,\mu):t\in[t_1-lh,t_1),\quad z(t_1,\mu),u(t)\in U\}$$

is dense in $\mathcal{M}_2^{n_1+n_2}$.

Definition 3. If there exists a number $\mu^*\in(0,\mu^0]$ that SPLTISD (1), (2) is controllable for any $\mu\in(0,\mu^*]$, we say that controllability is robust with respect to $\mu\in(0,\mu^*]$.

Define the matrix-valued operator and matrix:

$$\mathsf{A}_\mathsf{s}(e^{-ph})\equiv\mathsf{A}_1(e^{-ph})-A_2A_4^{-1}\mathsf{A}_3(e^{-ph}),\quad B_s\equiv B_1-A_2A_4^{-1}B_2,$$

and the sets of complex numbers:

$$\sigma_s\equiv\{\lambda\in C:\det[\lambda E_{n_1}-\mathsf{A}_\mathsf{s}(e^{-\lambda h})]=0\},\quad \sigma_f=\{\lambda\in C:\det[\lambda E_{n_2}-A_4]=0\}.$$

Theorem 1. *Let*

1) $\operatorname{rank}[B_s, \mathsf{A}_\mathsf{s}(z)B_s, \ldots, \mathsf{A}_\mathsf{s}{}^{n_1-1}(z)B_s] = n_1$ *for some* $z \in C$;

2) $\operatorname{rank}[\lambda E_{n_1} - \mathsf{A}_\mathsf{s}(e^{-\lambda h}), B_s] = n_1 \ \ \forall \lambda \in C: \ e^{-\lambda h} = z, \ \operatorname{rank}[B_s, \ldots, \mathsf{A}_\mathsf{s}{}^{n_1-1}(z)B_s] < n_1$;

3) $\operatorname{rank}[B_2, \mathsf{A}_4 B_2, \ldots, \mathsf{A}_4{}^{n_2-1}B_2] = n_2$,

then the SPLTISD (1), (2) is complete controllable for all sufficient small $\mu \in (0, \mu^0]$.

Sketch of the proof. Similar to [1] by using a non-degenerate decoupling transformation, we transform the original SPLTISD (1) into an equivalent system with separated motions.

For a fixed value of the parameter $\mu \in (0, \mu^0]$, the conditions of complete controllability from [2] are applicable to the resulting decoupled system. Then taking into account the preservation of the fullness of the rank of matrices with small additive perturbations, we have the sufficient conditions for the complete controllability of the SPLTISD (1) far all sufficient small $\mu \in (0, \mu^0]$:

$$\operatorname{rank}\begin{pmatrix} \lambda E_{n_1} - \mathsf{A}_\mathsf{s}(e^{-\lambda h}) & 0_{n_1 \times n_2} & B_s \\ 0_{n_2 \times n_1} & \mu\lambda E_{n_2} - A_4 & B_2 \end{pmatrix} = n_1 + n_2 \quad \forall \lambda \in C. \tag{3}$$

Taking into account the properties of the spectrum of the original SPLTISD (1) for a small values of $\mu > 0$ [1], we see that the conditions (4) follows from the conditions

$$\operatorname{rank}[\lambda E_{n_1} - \mathsf{A}_\mathsf{s}(e^{-\lambda h}), B_s] = n_1, \quad \lambda \in \sigma_s, \tag{4}$$

$$\operatorname{rank}[\lambda E_{n_2} - A_4, B_2] = n_2, \quad \lambda \in \sigma_f. \tag{5}$$

The relationship of the conditions (4), (5) and the conditions 1)–3) of the Theorem 1 prove the validity of the Theorem 1.

Theorem 2. *Let the conditions of the Theorem 1 are fulfilled and*

$$\operatorname{rank}\{A_{1l} - A_2 A_4^{-1} A_{3l}, B_s\} = n_1.$$

Then the SPLTISD (1), (2) is controllable in the space $\mathcal{M}_2^{n_1+n_2}$ *for all sufficient small* $\mu \in (0, \mu^0]$.

Theorem 2 is proved similarly to the proof of the Theorem 1 on the basis of conditions of controllability in the space $\mathcal{M}_2^{n_1+n_2}$ from [3].

Note, that the sufficient conditions of the Theorems 1 and 2 do not depend on a singularity parameter μ and are valid for all its sufficiently small values, i.e. robustly with respect to this parameter. The conditions have a parametric, rank form and are expressed in terms of the controllability conditions of two systems of a lower dimensions the the original one.

Acknowledgement. The work is partially supported by the Education Ministry of the Republic of Belarus, the state program of scientific research Convergence–2020, code of task 1.3.02.

References

1. Tsekhan O. B. *Decoupling transformation for linear stationary singularly perturbed system with delay and its applications to analysis and control of spectrum* // Bull. of the Yanka Kupala State University of Grodno. Ser. 2. Math. 2017. V. 7. № 1. P. 50–61 (Russian).

2. Marchenko V. M. *To controllability of linear systems with aftereffect* // Dokl. AN SSSR. 1977. V. 236. № 5. P. 1083–1086 (Russian).

3. Minyuk S. A., Lyakhovets S. N. *On a problem of controllability for systems with multiple delays* // Bull. of the Academy os Sciences of Byelorussian SSR. Ser. Fiz.-math. Sci. 1980. № 2. P. 12–17 (Russian).