ON THE SOLVABILITY OF PERIODIC PROBLEM FOR FOURTH ORDER SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS SPECIAL TYPE

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On the domain $\Omega = [0,T] \times [0,\omega]$ we consider a periodic problem for the system of partial differential equations of fourth order in following form

$$\frac{\partial^4 u}{\partial x^2 \partial t^2} = A(t,x)\frac{\partial^2 u}{\partial x^2} + B(t,x)\frac{\partial^2 u}{\partial t^2} + C(t,x)\frac{\partial^2 u}{\partial x \partial t} + f(t,x), \tag{1}$$

$$u(0,x) = u(T,x), \quad x \in [0,\omega], \tag{2}$$

$$\left.\frac{\partial u(t,x)}{\partial t}\right|_{t=0} = \left.\frac{\partial u(t,x)}{\partial t}\right|_{t=T}, \quad x \in [0,\omega], \tag{3}$$

$$u(t,0) = \psi_1(t), \quad t \in [0,T], \tag{4}$$

$$\left.\frac{\partial u(t,x)}{\partial x}\right|_{x=0} = \psi_2(t), \quad t \in [0,T], \tag{5}$$

where $u(t,x) = col(u_1(t,x), u_2(t,x), \ldots, u_n(t,x))$ is unknown function, the $n \times n$-matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, and n-vector function $f(t,x)$ are continuous on Ω, the the n-vector functions $\psi_1(t)$, $\psi_2(t)$ are twice continuously differentiable on $[0,T]$.

The compatibility conditions of initial data is valid:

$${}_1(0) = \psi_1(T), \quad \dot{\psi}_1(0) = \dot{\psi}_1(T), \qquad {}_2(0) = \psi_2(T), \quad \dot{\psi}_2(0) = \dot{\psi}_2(T).$$

Consider also auxiliary periodic problem

$$\frac{\partial^2 v}{\partial x \partial t} = A(t,x)\int\limits_0^t \frac{\partial v(\tau,x)}{\partial x}\,d\tau + B(t,x)\int\limits_0^x \frac{\partial v(t,\xi)}{\partial t}\,d\xi + C(t,x)v + g(t,x), \tag{6}$$

$$v(0,x) = v(T,x), \quad x \in [0,\omega], \tag{7}$$

$$v(t,0) = \dot{\psi}_2(t), \quad t \in [0,T], \tag{8}$$

where $v(t,x) = \mathrm{col}\,(v_1(t,x), v_2(t,x), \ldots, v_n(t,x))$ is unknown function, n"=vector function $g(t,x)$ are continuous on Ω.

Periodic and nonlocal problems for second order system of hyperbolic equations were studied in [1, 2].

Theorem. *Let the periodic problem for system of integro-differential equations hyperbolic type (6)–(8) unique solvable.*

Then periodic problem for system of partial differential equations of fourth order (1)–(5) has an unique classical solution.

The proof is proved analogously by proof of Theorem 1 in [3].

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