# MULTIDIMENSIONAL INTEGRAL TRANSFORMS WITH THE GAUSS HYPERGEOMETRIC FUNCTION IN THE KERNELS IN THE WEIGHTED SPACE OF SUMMABLE FUNCTIONS

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Two multidimensional integral transforms

$$ {}_1I^{c}_{\sigma,\omega}(a,b)f(\mathbf{x}) = \mathbf{x}^{\sigma}\int\limits_0^{\mathbf{x}} \frac{(\mathbf{x}-\mathbf{t})^{c-1}}{\Gamma(c)} F\left(a,b;c;1-\frac{\mathbf{x}}{\mathbf{t}}\right)\mathbf{t}^{\omega} f(\mathbf{t})\,d\mathbf{t} \quad (\mathbf{x}>\mathbf{0}); \tag{1} $$

$$ {}_1I^{c}_{\sigma,\omega}(a,b)f(\mathbf{x}) = \mathbf{x}^{\sigma}\int\limits_0^{\mathbf{x}} \frac{(\mathbf{x}-\mathbf{t})^{c-1}}{\Gamma(c)} F\left(a,b;c;1-\frac{\mathbf{t}}{\mathbf{x}}\right)\mathbf{t}^{\omega} f(\mathbf{t})\,d\mathbf{t} \quad (\mathbf{x}>\mathbf{0}) \tag{2} $$

are studied. Here $\mathbf{x}=(x_1,x_2,\ldots,x_n)\in\mathbb{R}^n$, $\mathbf{t}=(t_1,t_2,\ldots,t_n)\in\mathbb{R}^n$, $\mathbb{R}^n$ Euclidean $n$-space; $a=(a_1,a_2,\ldots,a_n)$, $b=(b_1,b_2,\ldots,b_n)$, $c=(c_1,c_2,\ldots,c_n)\in\mathbb{R}^n$, $0<c_j<1$ $(j=1,2,\ldots,n)$; $\sigma=(\sigma_1,\sigma_2,\ldots,\sigma_n)\in\mathbb{R}^n$; $\omega=(\omega_1,\omega_2,\ldots,\omega_n)\in\mathbb{R}^n$; $(\mathbf{x}-\mathbf{t})^{c-1}=\prod_{j=1}^n(x_j-t_j)^{c_j-1}$; $\int_0^{\mathbf{x}}=\int_0^{x_1}\int_0^{x_2}\cdots\int_0^{x_n}$; the expression $\mathbf{x}\geqslant\mathbf{t}$ means $x_1\geqslant t_1$, $x_2\geqslant t_2$, $\ldots$, $x_n\geqslant t_n$; $d\mathbf{t}=dt_1{\cdot}dt_2{\cdots}dt_n$ [1], [2]; $F(a,b;c;\mathbf{z})$ is a function of the form: $F(a,b;c;\mathbf{z})=\prod_{j=1}^n {}_2F_1(a_j,b_j;c_j;z_j)$, ${}_2F_1(a_j,b_j;c_j;z_j)$ $(j=1,2,\ldots,n)$ are the Gauss hypergeometric functions [3].

Our report is devoted to the study of tranforms (1) and (2) in the weighted spaces $\mathfrak{L}_{\overline{\nu},\overline{2}}$ summabe functions $f(\mathbf{x})=f(x_1,x_2,\ldots,x_n)$ on $\mathbb{R}^{\mathbf{n}}_{+\ldots+}=\{\mathbf{x}:\mathbf{x}\in\mathbb{R}^{\mathbf{n}}|\mathbf{x_1}>\mathbf{0},\ x_2>0,\ \ldots,\ x_n>0\}$, such that:

$$ \|f\|_{\overline{v},\overline{2}}=\left\{\int\limits_{R^1_+}x_n^{v_n\cdot 2-1}\left[\cdots\left[\int\limits_{R^1_+}x_2^{v_2\cdot 2-1}\left[\int\limits_{R^1_+}x_1^{v_1\cdot 2-1}|f(x_1,\ldots,x_n)|^2\,dx_1\right]dx_2\right]\cdots\right]dx_n\right\}^{1/2}<\infty \tag{3} $$

$(\overline{2}=(2,\ldots,2),\ \overline{v}=(v_1,\ldots,v_n)\in\mathbb{R}^n,\ v_1=v_2=\ldots=v_n)$.

Our investigations are based on representations of Equations (1) and (2) via the modified G-transform of the form

$$ (\mathrm{G}_{\sigma,\kappa}f)(\mathbf{x})=\mathbf{x}^{\sigma}\int\limits_{\mathbf{0}}^{\mathbf{x}}\mathrm{G}^{\mathbf{m},\mathbf{n}}_{\mathbf{p},\mathbf{q}}\left[\mathbf{xt}\,\middle|\,\begin{matrix}(\mathbf{a}_i)_{1,p}\\(\mathbf{b}_j)_{1,q}\end{matrix}\right]\mathbf{t}^{\kappa}f(\mathbf{t})\,d\mathbf{t} \quad (\mathbf{x}>0), \tag{4} $$

where $\mathbf{m}=(m_1,m_2,\ldots,m_n)\in\mathbb{N}_0^n$ and $m_1=m_2=\ldots=m_n$; $\mathbf{n}=(\overline{n}_1,\overline{n}_2,\ldots,\overline{n}_n)\in\mathbb{N}_0^n$ and $\overline{n}_1=\overline{n}_2=\ldots=\overline{n}_n$; $\mathbf{p}=(p_1,p_2,\ldots,p_n)\in\mathbb{N}_0$ and $p_1=p_2=\ldots=p_n$; $\mathbf{q}=(q_1,q_2,\ldots,q_n)\in\mathbb{N}_0^n$ and $q_1=q_2=\ldots=q_n$ $(0\leqslant\mathbf{m}\leqslant\mathbf{q},\ 0\leqslant\mathbf{n}\leqslant\mathbf{p})$; $\mathbb{N}=\{1,2,\ldots\}$, $\mathbb{N}_0=\mathbb{N}\bigcup\{0\}$, $\mathbb{N}_0^n=\mathbb{N}_0\times\mathbb{N}_0\times\ldots\times\mathbb{N}_0$; $\sigma=(\sigma_1,\sigma_2,\ldots,\sigma_n)\in\mathbb{C}^n$; $\kappa=(\kappa_1,\kappa_2,\ldots,\kappa_n)\in\mathbb{C}^n$; the function

$$ \mathrm{G}^{\mathbf{m},\mathbf{n}}_{\mathbf{p},\mathbf{q}}\left[\mathbf{z}\,\middle|\,\begin{matrix}(\mathbf{a}_i)_{1,p}\\(\mathbf{b}_j)_{1,q}\end{matrix}\right]=\prod_{k=1}^n\mathrm{G}^{m_k,\overline{n}_k}_{p_k,q_k}\left[z_k\,\middle|\,\begin{matrix}(a_{i_k})_{1,p_k}\\(b_{j_k})_{1,q_k}\end{matrix}\right] $$

is the product of the G-functions $G^{m_k,\overline{n}_k}_{p_k,q_k}[z_k]$ $(k = 1, 2, \ldots n)$ [4]. Mapping properties such as the boundedness, the range, the representation and the inversion of the considered transforms (4) and (1), (2) in the weighted space (3) are established.

**References**

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