INVESTIGATION OF THE DISCONTINUITY STRUCTURE IN THE PROBLEM OF A STREAM AT AN INCLINE

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In this paper we give an example which demonstrates that when constructing a solution with discontinuities, it is necessary to consider the structure of these discontinuities. If the discontinuities structure does not exist then the constructed solution is not valid.

Let us consider a flow at an incline, e.g., a snow avalanche. The continuity and momentum equations for the flow thickness (depth) h and depth-averaged velocity v are

$$\frac{\partial h}{\partial t}+\frac{\partial hv}{\partial x}=0, \quad \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=f-\frac{1}{h}\frac{\partial \mathrm{g}\cos\theta h^2}{\partial x}, \quad f=\mathrm{g}\sin\theta-\frac{\tau(h,v)}{\rho h} \tag{1}$$

Here x is coordinate down the slope, θ is the slope angle, g – gravity acceleration, $\tau(v,h)$ – friction at the bottom per unit area. The equations (1) are valid in the domain $0\leqslant x\leqslant X_f$; $X_f(t)$ is the coordinate of the leading edge of the flow. In front of the flow there is a static layer of material (e.g., snow). At the leading edge this material is destructed and entrained by the flow – the leading edge is a destruction front. Mass and momentum conservation laws at the destruction front are

$$\bar{h}(w-\bar{v})=wh_0=Q, \quad Q\bar{v}=\frac{\mathrm{g}\cos\theta\bar{h}^2}{2}-\frac{\sigma_* h_0}{\rho} \tag{2}$$

Here w is the front speed, $\bar{h}$, $\bar{v}$ are the depth and velocity at the front, h_0, σ_* are the depth and compression strength of the layer entrained by the flow. During motion the length of the stream increases. At large time the stream can be conventionally divided into two zones: a long zone I with a large longitudinal scale, and a relatively narrow zone II adjacent to the destruction front [1]. In zone I it is possible to neglect derivatives over x and t in the momentum equations (kinematic waves theory [2]) to obtain $\sin\theta-\tau(v,h)/(\rho h)=0$. This implies that $v=V(h)$. Substituting this relation into continuity equation reduce the problem to solution of one equation for h

$$\frac{\partial h}{\partial t}+a\frac{\partial h}{\partial x}=0, \quad a(h)=\frac{\partial hV}{\partial h} \tag{3}$$

In this approach, a narrow zone II is replaced by a kinematic jump. The only conservation law on this jump is

$$h_1(w-V(h_1))=h_0 w \tag{4}$$

where h_1 is the flow depth behind the kinematic jump. The Lax evolutionarity condition for this jump is $a(h_1)\geqslant w$. The equations (3), (4) together with the initial conditions and the evolutionarity condition for the jump, are sufficient to calculate the flow front speed and the distributions of velocity and depth in zone I. However, to be sure that the obtained solution is valid we should prove the existence of solution in zone II which describes the structure of the kinematic jump. The flow parameters in zone II are governed by equations (1), (2); the solution should be stationary in coordinate system moving with the velocity w; it should be possible to link the solutions in zones I and II. It was found that an answer depends on the value of the strength σ_*. If $\sigma_* \geqslant \hat{\sigma}$ then the structure of the kinematic jump for which w satisfies (4) exists. If $\sigma_* < \hat{\sigma}$ then the structure of the kinematic jump exists only at $h_1 = \bar{h}$. The latter is a condition at the kinematic jump additional to (4). The evolutionarity condition now reads $w > a(h_1)$. It means that the solution obtained without investigation of the jump structure is not valid at $\sigma_* < \hat{\sigma}$.

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References

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