Strength calculations for cylindrical transmissions with compound intermediate rolling elements

Mikhail E. Lustenkov

Testing Laboratory of Mechanical Drive Systems,
Belarusian-Russian University,
Mira av, 43, Mogilev, 212000, Belarus
Email: lustenkov@yandex.ru

Abstract: The paper describes the design and examines the principle of operation of the transmission with intermediate rolling elements. The transmission has small dimensions in the radial direction and good assembly properties. It comprises compound rollers, and periodic slots are formed by several cams. The number of rolling elements involved in the load transfer has been found. The dependences for strength calculation of the transmission and recommendations on the choice of its basic geometric parameters are given.

Keywords: transmission; speed reducer; rolling elements; periodic slots; cams; stress; strength calculations.

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Biographical notes: Mikhail E. Lustenkov has been an Associate Professor at the Belarusian-Russian University since 2003. He received his PhD degree at the Institute of Machines Reliability (IMR), Belarus, in 2001. He is also the Senior Vice-Rector of Belarusian-Russian University. His main research areas are mechanical transmissions, machine design and CAD-CAM.

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1 Introduction

Transmissions with intermediate rolling elements (TIREs) are a special class of mechanisms in which the load is transferred by a system of balls or rollers. A large number of different designs, which can be grouped according to certain characteristics (Stanovskoy et al., 2003), in particular, according to the direction of movement of mass centres of rolling elements in relative motion during operation of the transmission, have been developed. In the TIREs with axial movement of these mass centres their motion paths are located on cylindrical surfaces, so these TIREs can be called cylindrical transmissions or transmissions of cylindrical type. The TIREs of this group have a number of advantages: small dimensions in the radial direction, full dynamic balance, and high load capacity.

The task to develop calculation methods for these mechanisms is important, as in most known works the issues of strength analysis have not been considered. The structure and the fundamentals of kinematics of transmissions of cylindrical type are shown in

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Lehmann (1981). In Terada et al. (2007), the complex design of the cylindrical transmission with rolling elements and additional intermediate bodies is examined, its kinematic characteristics and the load design are analysed. In Bara (2006), the dynamic loads acting in the ball mechanism are investigated additionally.

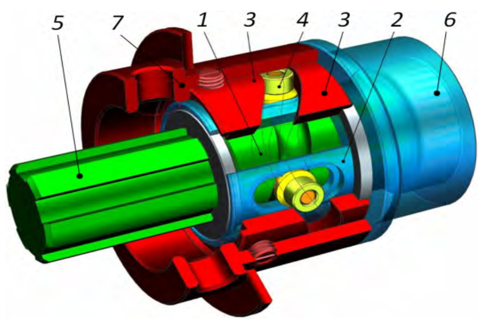
Fundamentals of these mechanisms theory and some issues of estimation of strength of cylindrical transmission parts are given in Ignatishchev (1983). The FEM analysis of speed reducers with TIREs of cylindrical type is given in Nam et al. (2013). However, in Lehmann (1981), Terada et al. (2007), Bara (2006), Ignatishchev (1983) and Nam et al. (2013) intermediate rolling elements in the transmissions under consideration are mainly balls. Small lengths of contact lines due to the limited dimensions of the ball reduce the load capacity of transmissions. The TIREs with solid rollers are examined in Pashkevich and Gerashchenko (1992). Like balls, solid rollers are simultaneously in contact with several parts, which inevitably results in their slipping in relation to one or more surfaces. Therefore, the disadvantage of most known mechanisms with intermediate rolling elements is their low efficiency.

Besides, there are technological difficulties in manufacturing periodic slots on inner cylindrical surfaces, and there is no possibility of wear compensation of these slots. We studied the design of the transmission (Lustenkov, 2010b), which is specific due to the use of rollers consisting of several components. The rollers move along the slots, each of them is made up of several parts. The objective of the research was to develop fundamentals of strength calculation of the parts of the transmission under consideration and to determine its basic geometric parameters based on the conditions of strength.

2 Transmission design and operation

The transmission is made up of two cams, the inner 1 and the outer 3 ones, and closed periodic slots are made on their cylindrical surfaces (Figure 1). The slots can be made on a solid cam (cam 1) and can be formed by face surfaces of two cams (cams 3). The rolling elements 4 move on these slots and along the axial slots of the cage 2. The inner cam is connected with the driving shaft 5, the cage is connected with the driven shaft 6, and the outer cams are fixed in the casing 7.

Figure 1 Design of TIREs of cylindrical type (see online version for colours)



The inner and outer cams and the cage form a three-link mechanism with the kinematics which is similar to the kinematics of a planetary gear. One link is a driving one, the second link is a driven one, and the third link is a fixed one during reducer (multiplier)

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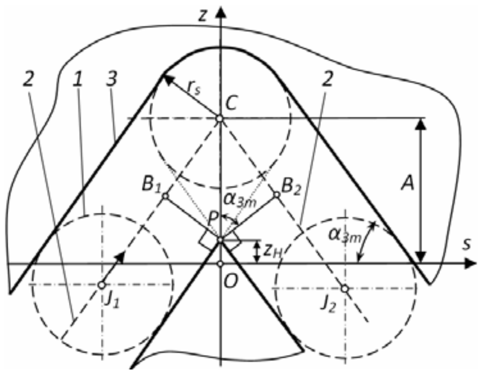
operating mode. Transmission ratio and speeds of the shafts can be determined by the Willis formula: $(\omega_1 - \omega_2) / (\omega_3 - \omega_2) = -Z_3 / Z_1$, where ω_1, ω_2, ω_3 represent angular velocities of the inner cam (link 1), the cage (link 2) and the outer cams (link 3) respectively; Z_1 and Z_3 are the number of periods of periodic slots on the inner and outer cams respectively.

The rolling elements are made as three-piece rollers. During operation of the transmission each of the three components of the roller is in contact with the corresponding working surface of the TIREs main links. Thus, the slipping friction is replaced by the rolling resistance having considerably lower energy losses. Due to the use of outer two-piece cams, mechanism adjustment and wear compensation of their working surfaces can be done by gradual movement of one of the cams along the transmission axis with a special adjustment mechanism.

3 Determining the number of load-transmitting rollers and effective forces in transmission

Let us see the diagram of passage of a rolling element (a cutter during cutting the slot) of the top of the slot by unfolding a fragment of the multi-period slot on plane (Figure 2).

Figure 2 Diagram of periodic slot segment formation



The centre of the rolling element 1 moves along the central curve 2. The central curve can be described by different functions. The most frequently used curve is a sinusoidal one which is described on plane (Figure 2) by the equation: $z_{1(3)} = A \cdot \sin(Z_{1(3)} \cdot s / R)$, where A is the amplitude of the central curve, R is the radius of the circle forming the cylindrical surface on which the mass centres of the rollers are located, the indices 1 and 3 are the indices of the central curve of the slot of the inner and the outer cams respectively.

To simplify subsequent calculations let us consider a sectional helical central curve. When the mass centre of the rolling element moves along the section of the curve J_1B_1, the rolling element itself can theoretically be in contact with both sides of the slot 3 (two-sided contact). According to the TIREs theory (Lustenkov, 2010b), in these sections the load is transmitted by the roller (ball). At the point B_1, when it moves from left to right (Figure 2), the contact with one side of the slot is broken and in the sections B_1C and CB_2 the rolling element makes a no-load run.

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At the same time, the total load on the transmission links is redistributed to the rolling elements, their centres of mass being on the working sections of the central curve. The parameter K_p, which is equal to the ratio of the length of working sections of the central curve to its total length, will determine the number of the rolling elements, whose mass centres are on the working sections of the central curve, i.e., which transmit the load, in relation to their total number. This parameter is determined by the following formula:

$$K_p = \frac{L - L_o}{L} = 1 - \frac{r_s \cdot \tan(\alpha_{3m}) \cdot \sin(\alpha_{3m})}{A} \quad (1)$$

where L is the overall length of the central curve, L_o is the total length of the sections of the central curve on which the rolling element makes a no-load run (B_1C), r_s is the radius of the rolling element, $\alpha_{1(3)m}$ is the average value of the slope angle of the one-period (multi-period) curve.

$$\alpha_{1(3)m} = \arctan\left(2 \cdot Z_{1(3)} \cdot A / (\pi \cdot R)\right) \quad (2)$$

The parameter K_p must be determined for the curve with a greater number of periods, in the case under consideration, for the slot formed by the outer cams ($Z_3 > Z_1$), since it has longer no-load sections. The average number of rolling elements, transmitting the load, can be estimated according to the following dependence: $n_p = K_p \cdot n$, where n is the total number of the rolling elements ($n = Z_1 + Z_3$).

The load capacity of the transmission and the strength of its parts are determined by the value of the normal force N_2, resulting from the contact of rolling elements with working surfaces of the cage and by the values of the forces N_1 and N_3, acting in the contact of rolling elements with the slots of the inner and outer cams respectively.

The force exerted on the side surface of the cage groove by one rolling element can be determined by the following dependence:

$$N_2 = \frac{M_2}{R \cdot n \cdot K_p \cdot K_n} \quad (3)$$

where M_2 is the torque on the output shaft, K_n is the factor of uneven load distribution along the power flows (depends on the accuracy of parts manufacturing).

In Lustenkov (2010a), the TIREs of spherical type are examined, and the forces acting on the main links of the transmission on the basis of the solutions of the system of equations of equilibrium of the transmission major links by using the principle of d'Alembert are determined. According to the given method, the values of the forces acting in the transmission under examination were obtained:

$$N_1 = \frac{N_2 \cdot \left(\frac{\cos(\alpha_{3m}) + f \cdot \sin(\alpha_{3m})}{\sin(\alpha_{3m}) - f \cdot \cos(\alpha_{3m})} - f \right)}{\cos(\alpha_{1m}) - f \cdot \sin(\alpha_{1m}) + \frac{\sin(\alpha_{1m}) + f \cdot \cos(\alpha_{1m})}{\sin(\alpha_{3m}) - f \cdot \cos(\alpha_{3m})} \cdot \left(\cos(\alpha_{3m}) + f \cdot \sin(\alpha_{3m})\right)} \quad (4)$$

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$$N_3 = \frac{N_2 - N_1 \cdot \left(\sin\left(\alpha_{1m}\right) + f \cdot \cos\left(\alpha_{1m}\right)\right)}{\sin\left(\alpha_{3m}\right) - f \cdot \cos\left(\alpha_{3m}\right)} \tag{5}$$

where f is the friction coefficient in the contacting pairs, based on the consideration of the processes of slipping and rolling.

4 Transmission strength calculations

Initial given data for the calculation are the maximum diameter of the reducer casing D_{max}, the transmission ratio u, the transmitted torque M_2, the transmission operation modes, the materials and heat treatment of parts, the type of the lubricant used. The values of the friction coefficient are taken according to the reference data.

The basic geometric parameters of the transmission are pre-calculated. The radius value R is taken on the basis of the D_{max} value. The slot on the inner cam is generally made as a one-period curve ($Z_1 = 1$). The number of the periods of the slot formed by the outer cams and the number of the rolling elements are determined by the transmission ratio value: $Z_1 = 1$, $Z_3 = u - 1$, $n = u$. The amplitude of the period curves A is determined on the basis of the condition of the least friction losses (Lustenkov, 2010b).

After determining the values of the forces according to formulas (2) to (4), which act on one roller, the torques acting on the TIREs cams are determined. The minimum diameter of the input shaft D_{1min} is calculated based on the condition of torsional strength. On the basis of the known diameters D_{1min} and D_{max}, the length of the roller, the inner and outer diameters of the cams, and, accordingly, the lengths of the contact lines are determined.

Let us consider the algorithms for the checking calculations of the TIREs parts. The compound roller of the TIREs under consideration comprises a rod with a diameter of d_{0s} and three bushings, one of which is made solid with the rod. The bushings with outer diameters of cylindrical surfaces d_{sj} (the index j is the index of the link, $j = 1 \ldots 3$) are in contact with respective main links of the transmissions. By analogy with the calculation of chain gears, bearing and shearing stress of the rods of the compound rollers of TIREs must be calculated. Bearing stress is determined by the following formula:

$$\sigma_{sj} = \frac{N_j}{2 \cdot k_{stj} \cdot r_{sj} \cdot l_{rj}} \tag{6}$$

where k_{stj} is the diameter factor of the roller rod ($k_{stj} = d_{0s} / d_{sj}$), l_{rj} is the length of the contact lines during interaction of the roller with the j^{th} part of the transmission, r_{sj} is the outer diameter of the roller bushing ($r_{sj} = 0.5.d_{sj}$).

The calculations according to formulas (2) to (4) show that the reaction N_2 has a maximum value, therefore the bearing stress is further determined in the contact of the roller bushing with the cage slot. Shear stress (with two shear surfaces) is determined according to the following formula:

$$\tau_N = \frac{N_2}{2 \cdot \pi \cdot k_{st2}^2 \cdot r_{s2}^2} \tag{7}$$

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At the stage of transmission design it is necessary to determine the diameters of the roller bushings. Let us express the maximum allowable force N_2 from formula (6), with the existing bearing stress σ_{s2} being replaced with permissible stress $[\sigma_s]$. Further, let us substitute the right side of dependence (2) for N_2 in this expression, the parameter K_p being replaced with equation (1). We will obtain a quadratic equation with a variable r_{s2}. It can be represented as a function:

$$fs_{\sigma}\left(r_{s2}\right) = S_1 \cdot r_{s2}^2 + r_{s2} - S_2 \tag{8}$$

where S_1 and S_2 are the coefficients of the quadratic equation.

$$S_1 = -\tan\left(\alpha_{3m}\right) \cdot \sin\left(\alpha_{3m}\right) / A,\ S_2 = M_2 / \left(2 \cdot R \cdot n \cdot K_n \cdot l_{r2} \cdot k_{st} \cdot \left[\sigma_s\right]\right) \tag{9}$$

Having done similar substitutions in expression (7), which determines the shear stress, and having made transformations, we obtain a cubic equation. Let us represent it in the form of a function as well:

$$fs_{\tau}\left(r_{s2}\right) = S_1 \cdot r_{s2}^3 + r_{s2}^2 - S_3 \tag{10}$$

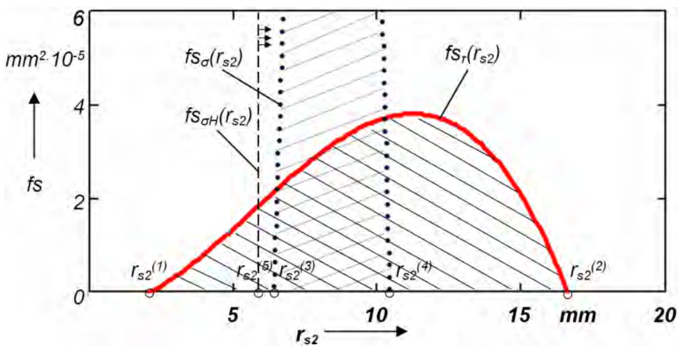
where S_3 is the coefficient of the cubic equation.

$$S_3 = M_2 / \left(2 \cdot R \cdot n \cdot K_n \cdot \pi \cdot k_{st}^2 \cdot \left[\tau_N\right]\right) \tag{11}$$

where $[\tau_N]$ is the permissible shear stress.

Let us represent the solution of the resulting quadratic and cubic equations graphically. Figure 3 shows a graph which allows determining the rational value of the outer diameter (radius) of the roller bushing which is in contact with the cage slots for the transmission with the following geometric and other parameters: R = 40 mm, A = 22 mm, Z_1 = 1, Z_3 = 4, u = 5, l_{r1} = 4.45 mm, l_{r2} = 4.5 mm, l_{r3} = 5.6 mm.

Figure 3 Graph for determining the minimum radius of the roller (see online version for colours)



The material used in all parts is tempered steel 45 (S 45 C). Thus: $[\sigma_s]$ = 25 MPa, $[\tau_N]$ = 72 MPa (Ivanov and Finogenov, 2006). The factor K_n is assumed to be 0.8, and the friction coefficient f = 0.05. The torque on the driven shaft M_2 = 63 Nm. At the rotation frequency of 1,000 r/min the transmitted power (ignoring the transmission efficiency) is 1.32 kW.

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The cubic equation represented by function (10) has the solutions $r_{s2}^{(1)}$ and $r_{s2}^{(2)}$, and the quadratic equation, represented in the form of function (8) has the solutions $r_{s2}^{(3)}$ and $r_{s2}^{(4)}$. The required value of the radius is within the intervals $[r_{s2}^{(1)}, r_{s2}^{(2)}]$ and $[r_{s2}^{(3)}, r_{s2}^{(4)}]$. The second interval is the determining one, as it is located inside the first one. The universality of this conclusion is proved by numerical calculations. The ratio $r_{s2}^{(3)} / r_{s2}^{(1)}$ for the transmission with the above parameters when the torque on the driven shaft M_2 changes in the range 40 … 100 Nm, is 2.0 … 2.4; with R = 60 mm and other parameters being equal, this range is 1.7 … 2.1. This shows that the minimum radius r_{s2} should be determined from the bearing stresses. It is necessary to choose the minimum value within the interval $[r_{s2}^{(3)}, r_{s2}^{(4)}]$, since the actual bearing stress is equal to allowable ones, and the parameter K_p is maximal.

It means that the material of the roller bushings will be used most efficiently and the acting loads will be minimal under these conditions. The required solution is one of the real roots of the quadratic equation, represented by function (8): $r_{s2}^{(3)}$. Thus, the minimum radius of the roller is calculated by the formula:

$$r_{s2}=\frac{A\cdot\left(1-\sqrt{1-\dfrac{2\cdot M_2\cdot\tan\left(\alpha_{3m}\right)\cdot\sin\left(\alpha_{3m}\right)}{R\cdot A\cdot n\cdot K_n\cdot l_{r2}\cdot k_{st}\cdot\left[\sigma_s\right]}}\right)}{2\cdot\tan\left(\alpha_{3m}\right)\cdot\sin\left(\alpha_{3m}\right)} \tag{12}$$

The contact strength calculation is based on the transformed Hertzian formula for determining maximum contact stress for the contact of flat and cylindrical surfaces of steel parts [Birger et al., (1993), p.531]. The calculation of the maximum contact stress σ_{Hj}, MPa, in each of the contacts of the rolling elements and the condition of the contact strength of the TIREs under consideration are written as follows.

$$\sigma_{Hj}=191,67\cdot\sqrt{\frac{N_j}{r_{sj}\cdot l_{rj}}},\ \text{MPa},\ \sigma_{H\max}=\max\left(\sigma_{Hj},\ j=1\ldots3\right)\le\left[\sigma_H\right] \tag{13}$$

where $[\sigma_H]$ is permissible contact stress.

The values of the radii of the bushings and the lengths of the contact lines (r_{sj}, l_{rj}) in expression (13) should be in millimeters.

The minimum radii of the outer surfaces of the roller bushings r_{sj}, mm, based on the condition of the contact strength are determined by the formula:

$$r_{sj}=36,74\cdot\frac{N_j}{\left[\sigma_H\right]^2\cdot l_{rj}} \tag{14}$$

The minimum radius of the outer surface of the roller bushing, which is in contact with working surfaces of the cage slots in the transmission with the above mentioned geometric parameters and loading parameters, is limited by the allowable contact stress and is determined in Figure 3 by the function $fs_{s\sigma H}(r_{s2})$.

The flexural stress calculation of the lobes of the outer cams for the TIREs is a checking one. If we neglect the curvature of the lobes in a plane which is perpendicular to the axis of the transmission, the known design model (Ivanov and Finogenov, 2006) of a gear tooth can be used for flexural strength calculation. Let us consider the lobe as a cantilever beam. We take the section at the lobe base where rounding begin (Figure 2) as

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a calculated one. The calculated flexural stress σ_F is determined on the stressed side of the lobe, according to the following formula, as the difference in tensile and compressive stress.

$$\sigma_F = \frac{N_3 \cdot \sin(\alpha_{3m}) \cdot h_f}{W_f} - \frac{N_3 \cdot \cos(\alpha_{3m})}{H_f} = N_3 \cdot \left(\frac{6 \cdot \sin(\alpha_{3m}) \cdot h_f}{l_{r3} \cdot s_f^{\ 2}} - \frac{\cos(\alpha_{3m})}{l_{r3} \cdot s_f} \right) \quad (15)$$

where W_f is the section modulus at bending, H_f is the sectional area of the lobe, h_f is the distance from the point of application of force N_3 to the dangerous section, s_f is the width of the lobe.

Let us see how the values of flexural stress change from the top point of the lobe (point P in Figure 2) to the assumed dangerous (weakest) section. Let us consider the expression for determining the flexural stress as the function of h_f. We take into consideration that the thickness of the lobe varies along its height as well $s_f = 2 \cdot h_f / \tan(\alpha_{3m})$. The height of the lobe varies from 0 to the value $h_{f\max}$, which is determined according to the dependence:

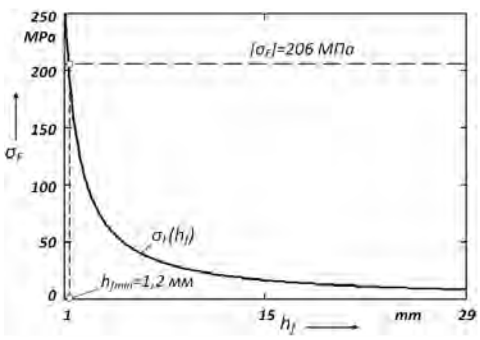
$$h_{f\,\max} = 2 \cdot A - r_{s3} \cdot \left(\frac{1}{\cos(\alpha_{3m})} \right) \quad (16)$$

The expression to determine the flexural stress (15) can be finally written as follows.

$$\sigma_F(h_f) = \frac{N_3 \cdot \sin(\alpha_{3m})}{2 \cdot l_{r3} \cdot h_f} \left(3 \cdot \tan^2(\alpha_{3m}) - 1 \right) \quad (17)$$

The results of the calculations with the above mentioned geometric parameters of the transmission and its loading parameters are shown in Figure 4.

Figure 4 Graph of changes in flexural stress depending on height of application of reaction



The maximum flexural stress occurs at the top of the lobe, which may cause its breakage. At the same time, the height of the lobe section $h_{f\min}$, where the calculated stress exceeds the allowable ones is negligible (Figure 4). This section can be cut off during manufacture. The height of the cut-off section is calculated by the formula:

$$h_{f\,\min} = \frac{N_3 \cdot \sin(\alpha_{3sm})}{2 \cdot l_{r3} \cdot [\sigma_F]} \left(3 \cdot \tan^2(\alpha_{3m}) - 1 \right) \quad (18)$$

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where $[\sigma_F]$ is the permissible flexural stress.

After determining the values $h_{f\min}$ it is necessary to specify the values K_p (formula (1)) and the values of all the forces acting in the transmission [formulas (2)–(4)].

In addition to the calculations presented, the calculation of normal flexural stress τ_{Ns} of the cage sections located between the slots in the dangerous sections, where the rounding of the slots begin, is made. The force N_2 in this calculation is applied in the centre of these sections. Also, the screw connections holding the cams are tested for shear and key connections are tested for bearing stress.

Let us see as an example the results of calculations of the stress acting in the TIREs parts with geometric parameters given in the explanations to Figure 3, depending on the value of the torque on the output shaft. The radius of outer cylindrical surfaces of all roller components was assumed to be equal $r_{s1} = r_{s2} = r_{s3} = 6$ mm. Tempered steel 45 was used as the material for manufacturing the cams, the cage and the roller components [yield strength $\sigma_t = 360$ MPa; Ivanov and Finogenov (2006, p.54), Table 1]. The periodic operation mode with occasional peak overloads was examined. The lubricant was the consistent substance (a mixture of graphite and semi-synthetic oil with hypoid additive), put during the assembly of reducer. We take the friction coefficient $f = 0.05$, subject to rolling and slipping.

Table 1 Values of stresses and parameter $h_{f\min}$ (see online version for colours)

M_2, *Н·м*	σ_s, *MPa*	$\tau_N \cdot$ *MPa*	$\sigma_{Hmax} \cdot$ *MPa*	$\tau_{Ns} \cdot$ *MPa***	$h_{fmin} \cdot$ *mm*
50	16.8	8.0	785.5	29.6	0.8
55	18.5	8.8	823.9	32.6	0.8
60	20.2	9.6	860.5	35.5	0.9
65	21.8	10.4	895.6	38.5	1.0
70	23.5	11.2	929.4	41.5	1.1
75	25.2	12.0	962.1	44.4	1.2
80	26.9	12.8	993.6	47.4	1.2
85	28.6	13.6	1,024.1	50.3	1.3
90	30.2	14.4	1,054.0	53.3	1.4
95	31.9	15.2	1,083.3	56.3	1.5*
100	33.6	16.0	1,111.2	59.2	1.5*

Notes: *The increase of $h_{f\min}$ to more than 1.4 mm is not advisable, as the value of the parameter K_p decreases significantly, and the normal reactions in the transmission increase.

**The flexural stress in the material of the cage between the cage slots was determined as the ratio of the maximum bending moment equal to $0.5 \cdot N_2 \cdot l_p$ (l_p is the length of the cage slots without rounding) to the section modulus, which depends on the diameter and the design of the roller bushing which is in contact with the cage.

The permissible bearing stress [as for movable key connections [Ivanov and Finogenov, (2006), p.94] was $[\sigma_s] = 25$ MPa. The permissible shear stress was determined by the formula $[\tau_N] = 0.2 \cdot \sigma_t = 78$ MPa, according to Ivanov and Finogenov (2006, p.55) (Table 1), as for the bolts placed without clearance at variable load. The permissible contact stress was calculated according to the expression $[\sigma_H] = 2.8 \cdot \sigma_t = 1{,}008$ MPa, as for the gears during the periodic operation mode with short-time single overloads. The permissible flexural stress was also determined by the calculation method for gearings described in Reshetov (1989, p.186): $[\sigma_F] = 2$ MPa.

Table 1 depending on M_2, gives the calculated values of the maximum bearing σ_s and shear τ_N stress acting on the rollers, the calculated values of the maximum contact stress $\sigma_{H\max}$ and of the maximum normal flexural stress acting in the cage τ_{Ns}, and those of the minimum required height of shear of the lobes of the outer cams to assure flexural resistance. The cells where the strength condition according to the corresponding criterion is violated are marked with inverse colour.

To check the kinematic characteristics of the transmission a prototype reducer was developed (Figure 5). The casing diameter is 82.0 mm, the prototype length is 155.0 mm, the transmission ratio is 5.0. The reducer underwent a laboratory bench run with the engine power of 1 kW.

Figure 5 Reducer prototype, (a) roller mechanism (b) reducer assembly

(a) (b)

5 Conclusions

The obtained expression (1) for determining the number of the rollers involved in the load transfer makes it possible to estimate the forces acting on the main links of the transmission depending on the geometric parameters. For the transmission with the parameters: $R = 20$ mm, $A = 10$ mm, with the roller radius $r_s \leq 3$ mm and the number of periods Z_3 increased from 1 to 5, the force N_2 is reduced almost by half. However, when the radius r_s is increased to 5 mm, and $Z_3 = 3$, the optimum of the force function (its minimum) is observed, the further increase of the number of periods of the race of the outer cam causes a sharp increase (nearly double, when $Z_3 = 6$) of the force value N_2, which reduces the load capacity of the transmission.

The roller radius increase definitely increases the values of the forces in the transmission, for large values of the number of periods ($Z_3 > 4$) this increase is more marked. Thus, for the TIREs designed to transfer power of up to 2 kW with the maximum casing diameter of up to 100 mm, the recommended value of transmission ratios is in the range from 1 to 5.

It was found that unlike the transmissions with solid rolling elements (Lehmann, 1981; Terada et al., 2007; Bara, 2006; Ignatishchev, 1983; Nam et al., 2013; Pashkevich and Gerashchenko, 1992), in the proposed design of the transmission with compound (three-piece) rollers the outer diameter of the rollers is determined by the bearing stress acting in the bushing-rod contact of the roller (Table 1). The next most important criterion is contact strength of the transmission parts. Let us express the parameter k_{st}

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from the condition of compressive strength [formula (6)] by replacing the actual bearing stresses σ_{s2} by their allowable values $[\sigma_s]$. By substituting the maximum permissible value of the force N_2, determined by the criterion of contact strength, into the resulting expression after the transformation we obtain $k_{st} = 1.363.10^{-5} \cdot [\sigma_H]^2 / [\sigma_s] = 0.55$. Provided that $k_{st} > 0.55$ (the diameter of the roller rod increases relative to the outer diameter of its bushings) the criterion of contact strength will be the main one. The flexural and shear stress calculations of the transmission parts are made as checking ones.

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