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ON NEW STUDENTS' RESULTS IN QUALITATIVE THEORY OF DIFFERENTIAL EQUATIONS AND APPLICATIONS

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This article presents some new students' results in qualitative theory of differential equations and its application. Previous review on this topic see in [1].

1. On Non-Power-Law Behavior of Blow-up Solutions to Emden–Fowler Type Higher-order Differential Equations (I. Astashova, M. Vasilev) [8].

For the equation

$$y^{(n)} = p_0\,|y|^k \operatorname{sgn} y,\ \ n \geq 2,\ \ k > 1,\ \ p_0 > 0, \tag{1}$$

we study blow-up solutions, i. e. those with $\lim_{x\to x^*-0} y(x) = \infty$. The origin of the considered problem is described in [2] (problem 16.4), and [3]. It was earlier proved for sufficiently large n (see [4]), for $n = 12$ (see [5]), for $n = 13$ and 14 (see [6]), and for $n = 15$ (see [7]), that there exists $k = k(n) > 1$ such that equation (1) has a solution with non-power-law behavior. Now this result is proved for arbitrary $n \geq 12$.

Theorem 1. *For any $n \geq 12$ there exists $k > 1$ such that equation (1) has a solution $y(x)$ with derivatives*

$$y^{(j)}(x) = (x^* - x)^{-\alpha - j}\, h_j(\,\log(x^* - x)\,), \qquad j = 0,1,\dots,n-1,$$

where $\alpha = \frac{n}{k-1}$ and h_j are periodic positive non-constant functions on $\mathbb{R}$.

2. On positive solutions of a two-point boundary value problem for a class of high-order nonlinear ordinary differential equations (K. Belikova) [9].

Consider a family of two-point boundary value problems for a class of nonlinear ordinary differential equations

$$y^{(n)} + x^m |y|^k = 0, \tag{2}$$

$$y(0) = y'(0) = \dots = y^{(j-1)}(0) = y^{(j+1)}(0) = \dots = y^{(n-1)}(0), \tag{3}$$

$$y^{(i)}(1) = 0, \tag{4}$$

where ≥ 2, $m > 0$, $k > 1$, $0 \leq i \leq j \leq n-1$.

A related initial-value problem is studied, namely,

$$z^{(n)} + x^m|z|^k = 0, \tag{5}$$

$$z(0) = z'(0) = \dots = z^{(j-1)}(0) = z^{(j+1)}(0) = \dots = z^{(n-1)}(0), \tag{6}$$

$$z^{(j)}(0) = A, \tag{7}$$

where $A > 0$ is an arbitrary number.

The results on the existence and uniqueness of a positive on (0, 1) solution to each boundary value problem of the family are given. Properties of solutions for the class of equations are studied and a representation of the solution to the boundary value problem (2)–(4) by means of a solution to the related initial-value problem (5)–(7) is obtained.

Lemma 1. *Let $z(x)$ be a maximally prolonged to the right solution to the initial-value problem (5)–(7). Then there exist uniquely defined points $x_0 \geq x_1 > \dots > x_j > 0$ such that, for all $l = 0, \dots, j$, $z^{(l)}(x) > 0$ on $(0, x_l)$, $z^{(l)}(x_l) = 0$, and $z^{(l)}(x) < 0$ on (x_l, x^*), where $x^* \leq +\infty$ is the right-hand end-point of the domain of z.*

Theorem 2. *There exists a positive on* (0, 1) *solution y to the two-point boundary value problem* (2)–(4). *The solution is uniquely defined by the formula $y(x) = Bz(Cx)$, where $z(x)$ is a solution of the related initial-value problem* (5)–(7), *$C = x_i$ with x_i given by Lemma* 1, *and B is defined by $B = C^{\frac{n+m}{k-1}}$.*

3. Mathematical models of epidemics in closed populations and their visualization via web application PhaPl (I. Astashova, V. Chebotaeva, A. Cherepanov) [10].

Two simple mathematical models of epidemics, SIRS (susceptible-infection-recovered) and SEIRS (susceptible-exposed-infection-recovered), are considered. Related systems of differential equations are built and studied. The equilibrium points of these systems are found. Then the lines of the stability-to-instability transition are found for both of the models. Authors showed that in diseases there are parameters that can be influenced, so that the epidemic does not turn into a cycle. The general case of these models was described in works before (see references in [10]). However, we add a visual representation by Web application PhaPl [11].

PhaPl is a software to study and plot phase portraits of autonomous systems of two differential equations on a plane. It automates all steps of the solution process: it finds equilibrium points, linearizes the system at each equilibrium point, finds eigenvalues and determines stability. It is easy to use and suitable for students and researchers. It has been deployed for teaching purposes at Moscow State University of Economics, Statistics, and Informatics (MESI), Lomonosov Moscow State University (MSU) since 2013 and in Plekhanov Russian University of Economics since 2015. The new version of PhaPl was produced as a web application with all computations performed on the client side.

Therefore, it is easy to host PhaPl on almost any web server.

4. A dynamic model of unemployment with migration and delayed policy intervention (V. Kuryshkina, M. Khachikyan) [12].

Authors consider the situation where governments observe the stock of migrants on their territory, along with unemployed numbers. The model is described by a nonlinear differential system with distributed delay. Our system had following variables: the number of unemployed persons, the number of employed persons, the number of newly created vacancies through government intervention, the total number of jobs created by market, the number of immigrants that become part of the labor workforce at destination. The time delay is incorporated in creating new vacancies thought government intervention that depends on the observation of past unemployment and immigration. Also all entrants to unemployment class are qualified to do any job. Authors study the local stability behavior of the non-negative equilibrium in the case of no distributed delay, and then they make the numeric simulation. They simulated the situation where there are no delays in the system, and both migration and unemployment are factored in the promotion of vacancies.

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