УДК 372.851

PROJECTS TO FACILITATE MATHEMATICAL LEARNING

N. HRITONENKO
Department of Mathematics, Prairie View A&M University
Prairie View, TX, USA
O. YATSENKO
The University of Texas Health Science Center at Houston
Houston, TX, USA

Mathematics is one of the most difficult disciplines. Mathematical courses require a strong background of prerequisites; students without a solid mathematical foundation may fall behind, lose interest in the topic, and fail the class. At the same time, every class has some students that grasp a concept quickly, potentially becoming bored if the explanation proceeds too slowly. Thus, delivering lectures to students with different levels of preparation while following a schedule is challenging. Warm-ups, bonus problems, and projects can easily be incorporated into the course's curriculum to simultaneously foster student crea-

tivity, enhance student understanding, challenge stronger students, and assist weak students in catching up. The following paper will discuss the incorporation of projects, presenting students' responses as a reflection of their challenges and benefits. Student responses are presented in *italics*.

Projects compliment the topics studied in a course by are meant to enhancing student understanding, increasing their mathematical ability, and «pushing» students to succeed. Types of projects range from presenting information about a topic to researching a problem, and some can be challenging.

This paper considers a sample project where students are required to:

(i) design a problem with a given solution;
(ii) solve the problem though different methods and describe these methods;
(iii) provide a comparative analysis of methods employed and obtained results;
(iv) write conclusions and emphasize a preferable method.

If such an individual project is assigned in an Advanced Mathematics course while discussing Laplace Transforms, then the function can be assigned as $a\sin(bx)$, where a is the month of a student's birthday and b is the date of their birthday. This eliminates «collaborative» work on the project. Students must design a nonhomogenous ODE of the second order and construct the initial value problem with the given function as its solution. Next, they must solve the initial value problem by at least two different ways, using methods studied in differential equations and employing Laplace Transforms.

Another example of a project given in an Optimization course is to design a word or story problem that has utilizes optimization with at least one variable equal to the student's birthday. This optimization problem is to be solved using at least two different methods.

The following questionnaires not only reflects students' opinions, but also emphasize the usefulness of these projects. *The project-assignment was actually fun. S*tudents *felt that the optimization project was very unique to say the least,* but *it is one of the most challenging projects that* they *have completed intellectually. One portion that was difficult was trying to implement an optimization problem into a story. S*tudents *have never been tasked to create a problem within a project usually it is the opposite, where* they are *tasked to solve a problem. From the examples in class as well as the homework, solving it was not an issue and* students initially *didn't think that making up a problem would be so difficult.*

Although it was interesting, students *also feel that there are a lot of vague or obscure ways to have completed the project, so it leaves a lot of room for error. S*tudents *found this project to be difficult, and worthy of a project. S*tudents *liked how* they *could create* their *own equation and try to solve it to become close to* their *roll number. S*tudents *spent many days making sure* their *project was acceptable and met all criteria.* They *worked on the project so much that* they *began to question* their *work and made several mistakes along the way.*

Students admit that they *have never worked so hard* but they *appreciate the project though and recommend it.* They admire how much they learned.

Creating a problem received very high evaluation. Students emphasize that it *makes them think and understand the methods better*, helps them to *gasp a concept better, strengthens mathematical skills, allows them to pick a problem they are comfortable with*. Although it is *challenging to think of problems on their own* it *allows them to be a scientist*. *Some methods have conditions that won't work properly. Then they have to start all-over*.

Some negative remarks state that it is *tedious at times to think of a problem*, and that students *don't know whether the problem is good, have to know the methods before making up a problem, and understand the importance of each method.* The last comments say everything: students must study and practice way before final exams come that will increase their final grade on the course.

Solving the problem by different ways *allows seeing different ways to work out a problem* and makes students think of *the entire concept being learned, their advantages and disadvantages*.

Students emphasize that they learn a lot during this *very time-consuming* task though *there is a chance to get two different answers and be confused.* While explaining the project, the instructor should ensure students that their grade will not be decreased for possibly incorrect solutions if properly explained. Students' explanations vary widely in explaining such inconsistencies: (i) *You solve the equation by two different methods and get two different solutions that are the same only at the initial point. The solutions are different because two different ways were used.* (ii) *It is evident that the results I have received for both methods don't match. I attributed this to the complexity of my problem.* More important is that the students thought, researched, and learned.

In general, students appreciate such creative projects because they *feel like an analyst of some sort,* realize that *some methods are easier to use than others,* are able to *determine which method is the best,* and *understand why and when one method is used instead of the other*. This is especially great practice before exams.

In a class of fifty or less students, a double-blind peer review evaluation can be implemented. In these, students submit two paper copies of their project, one with and one without their name. The nameless copy is given to another student from the same group for review. The student-reviewer reads the project and prepares their comments with possible corrections, if any. Two copies of the review, one with and one without their name, are turned in. The review without last name, instructor's comments, and grade is returned to the project's writer. This evaluation process is good preparation to students' future career.

Students rate this activity favorably, emphasizing its benefits. They mention that reviewing process *helps them refresh and review class material under study*. C*orrecting some of other students' mistakes* they *learn from others*. Reviews on their project helps them to *receive the honest opinion* and learn *what*

other people see in their projects and what they think about it. They *show ways of how to improve other projects.*

At the same time, some students are *not comfortable with another student's evaluation and grading*, highlighting that some reviews are *limited to peer knowledge* and prepared by *people that don't know what they are doing.* Despite these unfavorable comments, students generally admit that peer evaluation is a useful activity that significantly *improves their skills*. Pre-service teachers point out that peer-reviewing is *teaching and assessing at the same time* and that they *get some experience for their future grading papers*.

Incorporating projects and similar activities in classroom teaching brings intellectual challenges, strengthens mathematical preparation, enhances students' understanding of material, and significantly improves their final grades. Completing these projects gives students the opportunity to look at a problem from different directions, apply alternative approaches to solving it, and *learn more than they have for all* their *years of study*.